Unit 1
Lesson-1

PROPERTIES OF THE ATOMIC NUCLEUS

The objectives of the lesson are to explain the following:

1.1 Introduction

1.2 Nuclear size and its determinations

1.3 Theories of nuclear composition

1.4 Packing fraction

1.5 Mass Defect

1.6 Binding energy and its variations with the mass number

1.7 Magnetic dipole moment

1.8 Classical multipole moments for point charges

1.9 Electric Quadrupole moment

1.10 Summary.

1.1 Introduction: Since the atom is electrically neutral under normal conditions and the atoms of radioactive element are transformed into atoms of another element by emitting negatively or positively charged particles, hence it can be said that atoms are made up of equal number of positive and negative charges. The important questions then arose: 1. How many electrons were there in an atom ? and 2. How were the electrons and positive charges arranged in it ?. The scattering of X-rays shown that the number of electrons in an atom of the higher elements was equal to about half the atomic weight, except that in the atom of hydrogen. Thomson assumed that an atom consisted of a sphere of positively electricity of uniform density throughout of which was distributed an equal and opposite charge in the form of electrons. Thomson's theory came into conflict with the experiments of Rutherford and his collaborators on the scattering of α particles.. Rutherford had noticed that when alpha particles from a radioactive source fell on a photographic plate after penetrating a thin sheet of metal, the resulting trace was diffused sending fading off at the edges instead of being sharp. This spreading out of the particles on passing through thin layers of matter was called scattering . It was as a result of interaction with the atoms of the material through which they had passed. Rutherford and Geiger carried out experiments to count alpha particles with Geiger's gas counter. Geiger showed the small angle of alpha scattering and found nothing very unexpected. In view of J.Thomson model of the atom of appeared

possible that alpha particle scattering might be due to encounters with electrons. But Rutherford said Remembering that the mass, momentum and K.E of alpha particles are very large compared with the corresponding values of the electron. It does not seem possible that an alpha particle can be deflected through a large angle by a closed approach to an electron. Therefore, he concluded that the atom did not consist of a uniform sphere of positive electrification as supposed by Thomson, but that the positive charge was concentrated in a small region called the nucleus of the centre of the atom.

1.2 Nuclear size and its determination: In 1919 Rutherford observed that deviation from pure coulombs scattering anamalous scattering, was observable when alpha rays were scattered by the lightest elements, In these light elements, the closest distance of approach was of the order of $5 \times 10^{-15} m$. The distance of closest approach at which anomalous scattering begins was modified as the first measure of the nuclear radius. The shape of the nucleus is taken spherical because for a given volume this shape possesses the least surface and hence therefore provoke maximum short range binding forces between the nucleons(protons and neutrons) in the nucleus. Small asymmetries of the distributions of -ve charge are present in some metal as these nuclei exhibit high electric quadrupole moments. In most nuclei the ellipticity is only of the order of one percent. Thus we may suppose the protons are uniformly distributed inside the spherical nucleus.

There is an evidence that nuclear density ρ remains approximately constant over most of the nuclear volume and then decreases rapidly to zero. Thus means that nuclear volume is approximately proportional to the number of nucleons i.e. mass number 'A' . Hence radius of the nucleons $R \infty A^{\frac{1}{3}}$ *or* $R = R_0 A^{\frac{1}{3}}$

The methods of measuring nuclear radius are divided in to two main categories . One group of method is based on the study of the range of nuclear forces in which the nucleus is probed by a nucleon of light nucleus. Other group of methods studies the electric field and charge distributions of the nucleus in which the nucleus is probed by electron or muon(μ -meson).

1.3 Theories of nuclear composition: Rutherford's nuclear theory based on the scattering of alpha particles suggest the nucleus to be of compact structure. Natural radioactivity suggested that though the nucleus is compact,, it is capable of emitting several particles e.g. α, β, γ particles. The study of alpha and gamma ray spectra suggests discrete levels in the nucleus while beta ray spectra suggests a new particle neutrino. Researches in artificial transmutation of elements show that alpha particles , $p, n, e^{-} and \quad e^{+}$ should be present in the nucleus some how of other cosmic ray studies predict us another particle like 'meson' inside it. Some of the well established properties of atomic nuclei are:

1.All nuclei are +vely charged and the magnitude of the electronic charge is an integral multiple (z) of the proton charge e.

2. More than 99.9% of the mass of an atom is concentrated inside the tiny volume of the nucleus .

3.We assume that nuclei are spherical or nearly spherical in shape having radius 'R' is given by

$$R = R_0 A^{\frac{1}{3}}$$

4.The important correlation between R and A suggests that there is a universal density for nucleon matter density

$$\rho = \frac{mass}{volume} = \frac{A \times 1.66 \times 10^{-27} Kg}{\frac{4}{3}\pi\left(1.2 \times 10^{-15}\right)^3 Am^3} = 10^{17} Kg/m^3$$

$$Number \;\; of \quad nucleons/c.c. = \frac{Density}{Mass \;\; of \;\; a \;\; nucleon}$$

$$= \frac{10^{17}}{1.66 \times 10^{-27}} \approx 10^{44} nucleons/m^3$$

5.The nucleus is lightly bound system of the nucleons with a large potential energy. Several theories of nuclear compositions were put forward.

1.Proton-Electron theory: Prout suggested that all atomic weights were whole numbers and hence could be assumed as the integral multiple of the atomic weight of hydrogen.The slight change in the whole number was explained by the presence of two or more isotopes. This hydrogen theory confirms the presence of protons inside the atom. Since the electrons are

known to be emitted from some nuclei, it was natural to believe in the beginning that the electrons were the constituents of all nuclei. To account for the mass number of the nucleus whose atomic weight is nearly equal to the integer A, it was necessary to assume that nucleus would consist 'A' .But if this were the case the charge on the nucleus would be 'A', not equal to the atomic number'Z'. To remove this difficulty, it was assumed that in addition to 'A' protons the atomic nuclei would contain (A-Z) electrons. These would contribute a negligible amount to the mass of nucleus but would make the charge +z as required.

But this theory led to a number of contradictions with experiments and few of them are presented here

1. finite size
2. spin consideration
3. statistics
4. magnetic moment considerations
5. wave mechanical considerations
6. Compton wavelength
7. Beta decay
8. Electron neutrino pairs

The above points ruled out the existence of electron as semi permanent nuclei particles

2.Proton Neutron theory: The experimental discovery of the neutron led Heisenberg in 1932 to suggest that the nuclei might be composed of proton and neutrons rather than of protons and electrons. Thus for an atom $_zX^A$,the nucleus contains Z protons and (A-Z) neutrons. The total no. of particles inside the nucleus = A, the mass number. This nucleus is surrounded by Z electrons to make the atom electrically neutral . This theory avoids the failure of proton-electron theory.

1.4 Packing Fraction:-

The precision development in mass spectroscopy has shown that masses of isotopes are not whole numbers but they are very close to the whole number. Aston defined a new physical quantity known as packing fraction is expressed as

$$f = \frac{M - A}{A}$$

$$M = A(1 + f) \quad \text{..................1.1}$$

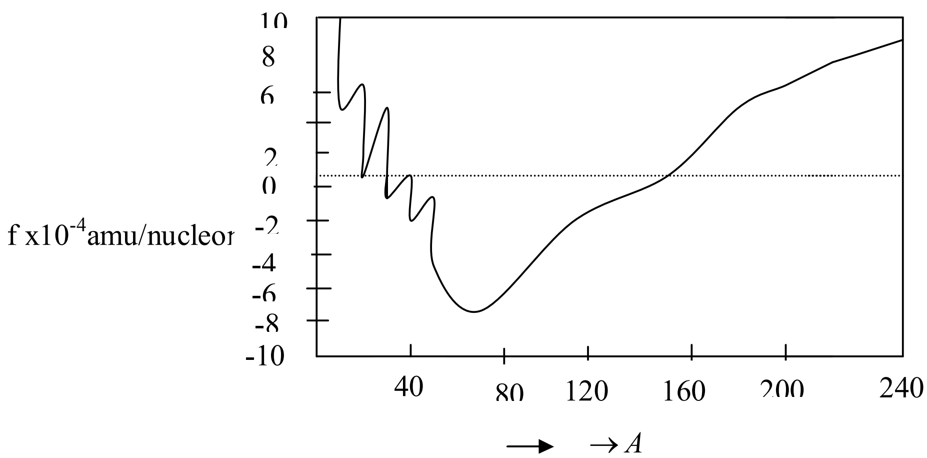

Fig:1. Packing fraction versus mass number

Where M is mass of the isotope and A is nearest whole number. The variation of packing fraction with mass number for large number of nuclides is shown in the fig(1). The curve is approximately smooth, f has high positive values for light elements.

It is zero for o^{16}. Now as A increases, packing fraction becomes negative pass through a flat minimum and finally becomes positive for A values for 180. It will be discussed that packing fraction is a useful quantity while explaining the stability of nuclides.

1.5 Mass Defect:-

Packing fraction is written as

$$f = \frac{\Delta M}{A} = \frac{\text{exp}\,erimental\ \ mass\ \ -\ \ whole\ \ number}{whole\ \ number}\text{...........................}1.2$$

In the beginning ΔM was termed as mass defect . This means that packing fraction is mass defect per nucleon. Some spectroscopists defined mass defect as (A-M) in place of (M-A).This introduces negative sign in the old definition i.e.

$$f = -\frac{mass\ defect}{A}$$

to avoid this confusion the expression for packing fraction relating the mass defect may be written as

$$f = \frac{mass\ \ difference}{A}\text{............................}1.3$$

It might, at first, be supposed that the mass of an atom should be the sum of the masses of its constituent particles i.e. protons and neutrons, electrons .But the experimentally measured mass of any stable atom is found to be less than the sum of the mass as of its constituents. The decrease in the mass is known as mass defect.

It is always true that actual mass never exceeds the added mass of its constituents. For a nuclide ${}_zX^A$, we write ${}_ZM^A$ for its actual atomic mass and $ZM_H\ \ and\ NM_n$ for the total mass of its constituents . The difference may be written as

$$\{ZM_H + (A-Z)M_n\} - {}_Z M^A = \Delta_Z M^A\text{.......................}1.4$$

This difference $\Delta_Z M^A$ is known as mass defect.

Example:1 Take ${}_2He^4$

It consists of 2 protons and 2 electrons and 2 neutrons. Therefore its expected mass should be

$$M_{He} = 2(M_H + M_n) \qquad (\because M_H = M_P + M_e)$$

Here all masses are expressed in atomic mass units . It is given that

Mass of hydrogen atom M_H =1.008145 amu

Mass of neutron M_n = 1.008986amu

Mass of Helium atom $M_{He} = 2(M_H + M_n)$

$= 2(1.008145 + 1.008986)$

$M_{He} = 4.034262$amu

But the experimentally measured mass value is 4.003873.

It is less by an amount 0.034amu

$$\Delta_z M^A = \{ZM_H + (A-Z)M_n\} -_z M^A$$

$$=4.03426 - 4.003873$$

$$\Delta_z M^A \approx 0.0304$$

Example:2 Argon $_{18}Ar^{40}$ No.of protons =18,No. of electrons = 18,

No. of Neutrons = 22

$$\Delta_z M^A = \{ZM_H + (A-Z)M_n\} -_z M^A$$

M_H = 1.008145amu

M_n = 1.008986amu

Experimental mass of Argon M_{Ar} =39.97505amu

$$\Delta m = \{18(1.008145) + (40-18)(1.008986)\} - 39.97505$$

$$= 40.344302 - 39.97505$$

$$\Delta m \approx 0.37 amu$$

1.6 BINDING ENERGY:

It is known that nucleus consists of large number of particles,namely neutrons and protons. Now the question is "what is the external agency that keeps these particles in a tightly bound structure inspite of the fact protons are positively charged and repel other protons ?" It turns out that particles should be held together by strong attractive forces. This idea is confirmed by the fact that when it is desired to break up a nucleus, i.e., work is done against the attractive forces, leaving aside, at present, the nature of the attracting forces we wish to discuss the origin of the attractive forces. When large amount of energy is supplied to a nucleus the constituents of nucleus are held apart. It means that total energy of the constituents (when they are at large distance) is greater than when they form nucleus. Conversely when the nucleons are brought together their total energy is less than the sum of the energies of its constituents when they are held apart. Now the natural question is what happens to the excess energy? Does it form the origin of attracting forces ?

It has been also seen that actual mass of nucleus is always less than the mass of its constituents. Where does the mass go?

The answers of the above questions become abundantly clear when all the questions are coordinated and we recall the famous Einstein relation

$E = mc^2$

where 'c' is the velocity of light in vacuum, m is any mass, and E is corresponding energy. The equation states that mass and energy are the manifestations of one same thing and one can be converted into other. Thus, we can say that mass defect $\Delta_z M^A$ appears as an equivalent amount of energy ΔE on forming a nucleus. ΔE is the energy released, due to the decrease of mass, when nucleus is formed by fusing together the requisite number of nucleons; alternately it is the energy required to separate the nucleons of nucleus. It is referred as the binding energy of the nucleus, B.E.

The binding energy of a nuclide $_Z X^A$ may be expressed as:

$$\begin{aligned} B.E &= \left(\Delta_z M^A\right)c^2 \\ &= \left\{ZM_H + (A-Z)M_n - {}_Z M^A\right\}c^2 \end{aligned}$$

The binding energy per nucleon will be

$$\frac{B.E}{A} = \frac{c^2}{A}\left\{ZM_H + (A-Z)M_n - {}_Z M^A\right\} \qquad \text{.......................1.5}$$

The variation of binding energy per nucleon with mass number is shown in fig.1.2

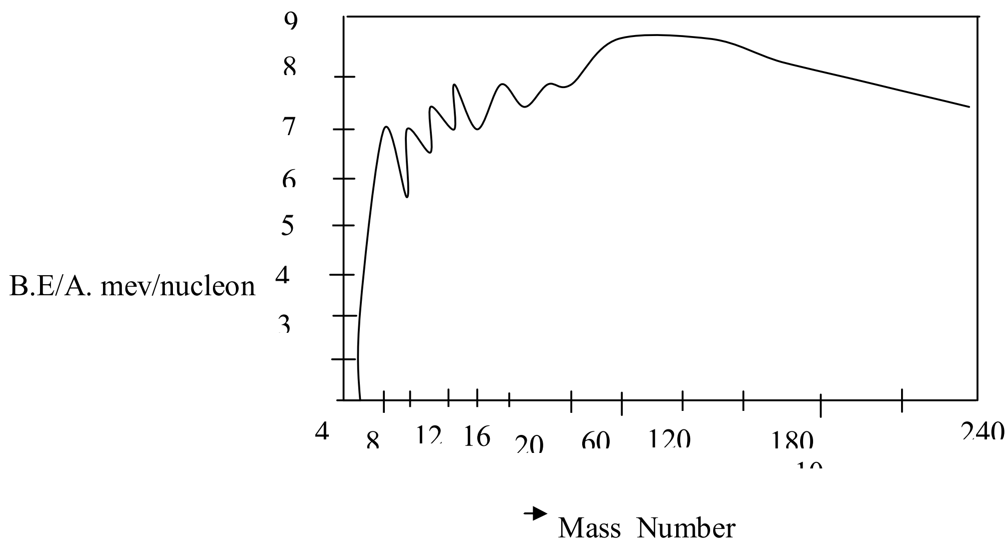

Fig:1.2. Binding Energy Curve

The variation of binding energyy per nucleon with mass number is shown in fig.2. The curve indicates that $\overline{B}$ is small for very light nuclides. It increases as A increases, reaches a maximum value of about 8.8Mev in the neighbourhood of A (= 50 $\sim$ 60). The maximum is quite flat and $\overline{B}$ is 8.4Mev at about A = 156. For higher mass number $\overline{B}$ decreases to about 7.6Mev for uranium . The curve also shows peaks for the nuclides of mass number 4,8,12,16,20. It is now desired to explain the above noted features in brief.

(a)Small binding energy in case of light nuclei:-

In the case of light nuclei, there are few nucleons and as a result, most of them are at the surface of the nucleus. This surface effect try to distrupt the nucleus and thereby reduces the binding energy of the nucleus.

(b)Occurrence of peaks in binding energy curve:-

The peaks occur at mass number 4,8,12,16,20.Let us take the example of ${}_2He^4 . {}_2He^4$ contains the maximum possible number of 1s nucleons, the four particles differing only with respect to their two possible spin orientations and two possible values of charge. This

brings out zero angular momentum for helium nucleus. It means that there is no centrifugal force to reduce the strength of binding forces and hence helium shows a tightly bound configuration associated with maximum value of B.E. The similar arguments hold good for other highly stable nuclei showing peaks.

(c) A constant binding energy per nucleon:-

If the energy is expressed in mass units, then binding energy expression is written as

$$B.E = ZM_H + (A-Z)M_n -_Z M^A$$
$$= AM_n + (M_H - M_n)Z -_Z M^A$$
$$\overline{B} = (M_n - 1) + \frac{Z}{A}(M_H - M_n) - \left(\frac{{}_Z M^A - A}{A}\right)$$
$$\overline{B} = (M_n - 1) + \frac{Z}{A}(M_H - M_n) - f ....1.5(a)$$

where f is packing fraction in atomic mass unit per nucleon. For the nuclide from A = 40 to 140, $\frac{Z}{A}$ has an average value of 0.46 in the same region

$$f = -6 \times 10^{-4} amu / nucleon.$$

Hence B = 0.008982 –(0.46)(0.000840) - f

= 0.0086 - f

= 8.5Mev/nucleon.

The average binding energy remain constant. Because(M_n-1) is the only predominating sterm and other terms are only correction terms.

(d) The decrease of binding energy per nucleon for high mass number:-

As the value of A further increases the coulomb repulsion increases. Furthermore, when A is sufficiently large nuclear forces are saturated and nuclear binding behaves like homopolar binding in chemical system. These two reasons are sufficient to account fall in binding energy per nucleon.

1.7 NUCLEAR MAGNETIC DIPOLE MOMENT:

Any charged particle moving in a closed path produces a magnetic field, which at large distances acts as due to magnetic dipole located at the current loop. The protons inside the nucleus are in orbital motion and therefore produce electric currents which produce extra nuclear magnetic fields. Each nucleon possesses an intrinsic magnetic moment which is parallel to its spin and is probably caused by the spinning of the nucleon. A spinning positive charge a magnetic field whose N-pole direction is parallel to the direction of spin. The magnetic moment is defined as positive in this case.

If a particle having a charge 'q' and mass 'm' circulates about a force center with a frequency 'ν'. The equivalent current i = qν. From Kepler's law of areas area swept $d\hat{A}$ in time dt by the particle is related with its angular momentum l as

$$\frac{d\hat{A}}{dt} = \frac{l}{2m} = \text{constant}$$

on integration over one period T, we get

$$\int d\hat{A} = \int_0^T \frac{\hat{l}}{2m} dt$$

$$\hat{A} = \frac{TI}{2m} \longrightarrow 1.6$$

Hence magnetic moment of a ring of current around an area of magnitude A is given by

$$\vec{\mu}_l = \mu_0 i\hat{A} = \mu_0 (q\upsilon)\left(\frac{T\vec{l}}{2m}\right) = \frac{q}{2m}\mu_0 \vec{l} \longrightarrow 1.7$$

Thus $\vec{\mu}_l$ and $\vec{l}$ are proportional. This relation is also valid in quantum mechanics. However, since the particles (electron, proton and neutron) possess a spin in addition to orbital angular momentum, experimentally it is found that the spin is also the source of a magnetic moment . Using q = e and a dimensionless correction factor g_s , we can write equation (1.7) as

$$\vec{\mu}_s = g_s\left(\frac{\mu_0 e}{2m}\right)\vec{s} \rightarrow 1.8$$

The factor g_s is different for the electron, proton and neutron. Similarly, we introduce a factor g_l and have

$$\vec{\mu}_l = g_l\left(\frac{\mu_0 e}{2m}\right)\vec{l} \longrightarrow 1.9$$

The total magnetic dipole moment '$\hat{\mu}$' is given as

$$\vec{\mu} = \vec{\mu}_s + \vec{\mu}_l = \left(\frac{\mu_0 e}{2m}\right)\left(g_s\vec{s} + g_l\vec{l}\right) \longrightarrow 1.10$$

For the nucleus of mass number A , magnetic dipole moment

$$\vec{\mu} = \frac{\mu_0 e}{2m}\left(\sum_{k=1}^{A} g_s\vec{s}_k + \sum_{k=1}^{Z} g_l\vec{l}_k\right) \longrightarrow 1.11$$

Since total angular momentum of the nucleus.

$$\vec{I} = \sum_{k=1}^{Z} \vec{l}_k + \sum_{k=1}^{A} \vec{s}_k \longrightarrow 1.12$$

$$\vec{\mu} = g\left(\frac{\mu_o e}{2m}\right)\vec{I} = g\left(\frac{\mu_o e\hbar}{2m}\right)\frac{\vec{I}}{\hbar} \longrightarrow 1.13$$

Where 'g' is the gyromagnetic ratio (g – factor) of the nucleus. It is the dimensionless ratio of the magnetic moment 'μ' in terms of $\mu_0 e\hbar/2m$ to the angular momentum in terms of ħ .

The magnetic dipole moment is measured in terms of nuclear magneton, defined as

$$\mu_N = \frac{\mu_o e\hbar}{2m_p} = \frac{\mu_0 e\hbar}{2m_n} \ldots\ldots\ldots\ldots 1.14$$

$$= 3.152 \text{ X } 10^{-8} \text{ ev} - \text{m}^2 \text{ / weber.}$$

The magnetic moment of even- odd and odd – even nuclei is due to only a single (unpaired) nucleon. If the odd nucleon is a proton $g_l = 1$, $g_s = g_p$ and if it is a neutron $g_l = 0$, $g_s = g_n$ .In this case S = ½ and $I = l + \frac{1}{2}$ or $l - \frac{1}{2}$

1.8 CLASSICAL MULTIPOLE MOMENTS FOR POINT CHARGES:-

In addition to its magnetic moment a nucleus may have an electric quadrupole moment. The property may be thought of as arising from an elliptic charge distribution in the nucleus. We will show that a nucleus can have a quadruple moment only if its angular momentum I ≥1.

1. The quadrupole moment would be zero if the +ve charge on a nucleus was distributed in a completely symmetrical spherical manner.
2. If the quadrupole moment would be +ve if the charge distribution, instead of being spherical, is drawn out in the direction of the spin axis in the form of prolate shape.
3. The quadrupole moment is -ve when the charge distribution is flattened about the spin axis in the form of oblate shape.

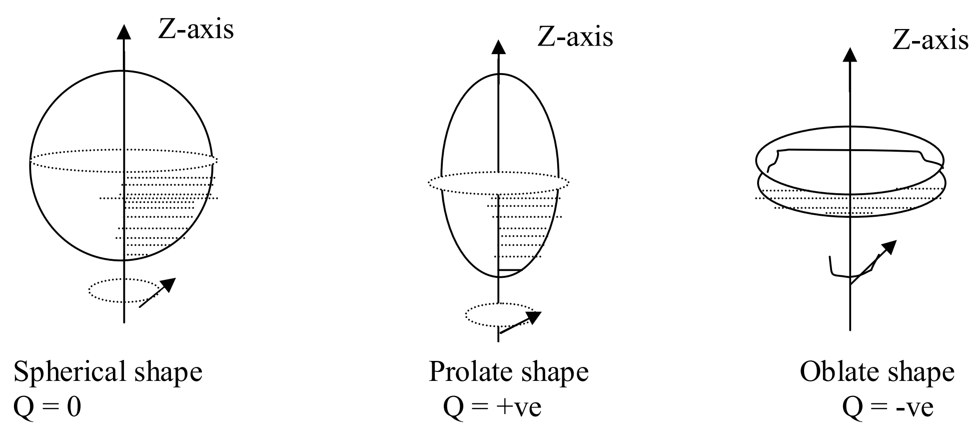

Fig.1.3 Illustration of quadrupole moment

As nucleus in a stationary state does not possess a dipole moment because the center of mass and center of charge can be assumed to coincide with each other. If the charge is not concentrated at the origin, then the potential at any point P, situated outside the nucleus

along the Z- axis and a distance ' R' from the center of the nucleus, due to a charge 'e' located at a point A(x,y,z) as shown in fig. is given by

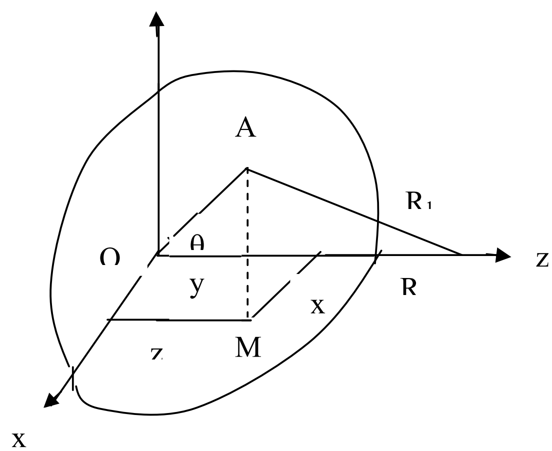

Fig. 1.4.Classicial picture for multipole moments

$$\phi_p = \frac{e}{4\pi\varepsilon_0 R_0} = \frac{e}{4\pi\varepsilon_0\left(R^2 + r^2 - 2RrCos\theta\right)^{1/2}}$$

$$= \frac{e}{4\pi\varepsilon_0 R\left(1 + \frac{r^2}{R^2} - \frac{2r}{R}Cos\theta\right)^{1/2}}$$

$$= \frac{e}{4\pi\varepsilon_0 R}\left(1 + \frac{r^2}{R^2} - \frac{2r}{R}Cos\theta\right)^{-1/2} \longrightarrow 1.15$$

In the Legendre polynomial we define the generating function

$$(1\text{-}2zx+z^2)^{-1/2} = \sum_{n=0}^{\infty} z^n P_n(x)\ldots\ldots\ldots\ldots\ldots\ldots 1.16$$

In the present case $\frac{r}{R} = z, \quad \cos\theta = x$

$$\left(1 + \frac{r^2}{R^2} - \frac{2r}{R} Cos\ \theta\right)^{-1/2} = 1 + \frac{r}{R} P_1(Cos\ \theta) + \frac{r^2}{R^2} P_2(Cos\ \theta) + \ldots\ldots\ldots \longrightarrow 1.17$$

From eqn 1.17 therefore, in the Legendre polynomial

we know that

$$\left.\begin{aligned} P_1(\cos\theta) &= \cos\theta \\ P_2(\cos\theta) &= \frac{3\cos^2\theta - 1}{2} \\ P_3(\cos\theta) &= \frac{5Cos^3\theta - 3Cos\theta}{2} \end{aligned}\right| \quad \text{-------------}1.18$$

Substituting eqn (1.18) in eqn (1.17)

$$\left(1 + \frac{r^2}{R^2} - \frac{2r}{R} Cos\theta\right)^{-1/2} = 1 + \frac{r}{R} Cos\theta + \frac{r^2}{R^2}\left(\frac{3Cos^2\theta - 1}{2}\right) + \frac{r^3}{R^3}\left(\frac{5Cos^3\theta - 3Cos\theta}{2}\right) + \ldots\ldots \longrightarrow 1.19$$

Substituting eqn (1.19) in eqn (1.15)

$$\phi_p = \frac{e}{4\pi\varepsilon_0 R}\left(1 + \frac{r}{R} Cos\theta + \frac{r^2}{R^2}\left(\frac{3Cos^2\theta - 1}{2}\right) + \frac{r^3}{R^3}\left(\frac{5Cos^3\theta - 3Cos\theta}{2}\right) + \ldots\ldots\right)$$

$$\phi_p = \frac{e}{4\pi\varepsilon_0 R} + \frac{er}{4\pi\varepsilon_0 R^2} Cos\theta + \frac{er^2}{4\pi\varepsilon_0 R^3}\left(\frac{3}{2} Cos^2\theta - \frac{1}{2}\right) + \ldots\ldots\ldots\ldots \longrightarrow 1.20$$

Here 'θ' is the angle between the r–direction and the z- axis. The term 'er' the product of charge and length, is referred to as the electric dipole moment. The term er^2, the product of charge and a surface area (r^2), is the electric quadrupole moment. The next term is called the octupole moment. Thus in the above equation the coefficients of 1/R, $1/R^2$, $1/R^3$, and so on are the monopole, z- component of the dipole moment, z- component of the quadrupole moment and z- component of octupole moment respectively exhibited at a point P(0,0,R).Thus even a single isolated charge exhibits a quadrupole and other moments if it is not located at the origin of co-ordinates.

Substituting Cos θ = z/r in the above equation for the coefficient of $1/R^3$, the effective classical quadrupole moment in the direction z exhibited at point P(0,0,R) is given by

$$q = \frac{1}{2} e\left(3z^2 - r^2\right) \ldots\ldots\ldots\ldots\ldots 1.21$$

The classical quadrupole moment would be eR^2 for a single proton situated at the nuclear radius R along the body axis. It would be -1/2 eR^2, for a single proton situated at the nuclear equator (z= 0 and r = R). Thus we can prove easily that the net quadrupole moment is zero for any spherically symmetric distribution of +ve charge in the nucleus. The nucleus will possess a net electric quadrupole moment, if there is one or more protons in addition to a symmetric distribution. It may have +ve or -ve value depending upon the position of the proton.

1.9 ELECTRIC QUADRUPOLE MOMENT:-

We now consider how the internal distribution of nuclear charge contributes to the effective moments. We place a nucleus having a charge density ρ(x y z) with its charge center at the origin. As the nucleus is surrounded by its orbital electrons hence an electrostatic potential Φ which originates from these electrons produces an electrostatic interaction energy, resulting from the interaction between φ and ρ. This energy is defined as

$$U = \int_v \rho(x,y,z)\phi(x,y,z)dv \longrightarrow 1.22$$

Taylor's series expansion of $f(x,y)$ is the neighbourhood of (a,b) is given

$$f(x,y) = f(a,b) + \frac{1}{1!}\left[(x-a)\frac{\partial}{\partial x} + (y-b)\frac{\partial}{\partial y}\right]_{\substack{x=a\\y=b}} f(a,b) + \frac{1}{2!}\left[(x-a)\frac{\partial}{\partial x} + (y-b)\frac{\partial}{\partial y}\right]^2_{\substack{x=a\\y=b}} f(a,b) + ...$$

$$+\frac{1}{n!}\left[(x-a)\frac{\partial}{\partial x} + (y-b)\frac{\partial}{\partial y}\right]^n_{\substack{x=a\\y=b}} f(a,b) + ...1.23$$

To express this energy in terms of the electric moments of the distribution, we expand the potential in a Taylor's series about the origin as

$$\phi(x,y,z) = \phi_0 + \left[\left(\frac{\partial\phi}{\partial x}\right)_0 x + \left(\frac{\partial\phi}{\partial y}\right)_0 y + \left(\frac{\partial\phi}{\partial z}\right)_0 z\right] + \left[\frac{1}{2}\left(\frac{\partial^2\phi}{\partial x^2}\right)_0 x^2 + \frac{1}{2}\left(\frac{\partial^2\phi}{\partial y^2}\right)_0 y^2 + \frac{1}{2}\left(\frac{\partial^2\phi}{\partial z^2}\right)_0 z^2\right] +$$
$$\left[\left(\frac{\partial^2\phi}{\partial x\partial y}\right)_0 xy + \left(\frac{\partial^2\phi}{\partial x\partial z}\right)_0 xz + \left(\frac{\partial^2\phi}{\partial y\partial z}\right)_0 yz\right] + ...\longrightarrow 1..24$$

Where the subscript '0' means that the quantity is evaluated at the origin. Inserting this value in eqn (1) with the idea that each of the derivatives in constant with respect to the variable of integration, we get

$$U = \phi_0 \int \rho dv + \left[\left(\frac{\partial \phi}{\partial x}\right)_0 \int x\rho dv + \left(\frac{\partial \phi}{\partial y}\right)_0 \int y\rho dv + \left(\frac{\partial \phi}{\partial z}\right)_0 \int z\rho dv \right] +$$

$$\left[\begin{array}{l} \frac{1}{2}\left(\frac{\partial^2 \phi}{\partial x^2}\right)_0 \int x^2 \rho dv + \frac{1}{2}\left(\frac{\partial^2 \phi}{\partial y^2}\right)_0 \int y^2 \rho dv + \\ \frac{1}{2}\left(\frac{\partial^2 \phi}{\partial z^2}\right)_0 \int z^2 \rho dv + \left(\frac{\partial^2 \phi}{\partial x \partial y}\right)_0 \int xy\rho dv + \left(\frac{\partial^2 \phi}{\partial x \partial z}\right)_0 \int xz\rho dv + \left(\frac{\partial^2 \phi}{\partial z \partial y}\right)_0 \int zy\rho dv + \ldots\ldots.. \end{array} \right] +$$

+ higher order terms ………..(1.25)

The first term gives simply the interaction energy of a point charge (mono pole). The three terms in the first bracket give the energy of a dipole. The six terms in the second bracket are the quadrupole energy terms. The above relation can be written in tensor form as

$$U = \phi_0 \int \rho dv + \left(\frac{\partial \phi}{\partial x_i}\right)_0 \int x_i \rho dv + \frac{1}{2}\left(\frac{\partial^2 \phi}{\partial x_i \partial x_j}\right)_0 \int x_i x_j \rho dv + \ldots\ldots\ldots\ldots\ldots\ldots\ldots \longrightarrow 1.26$$

Here integrals are the various moments of the distribution.

Let us now discuss quadrupole energy terms. For an ellipsoid of rotations, because of symmetry, the three integrals involving the cross products xy, yz and xz vanish. When the z-axis is the symmetry axis the integral over y^2 gives the same result as the integral over x^2.

i.e. $x^2 = y^2$

Thefore, quadrupole interaction energy is

$$\Delta U_2 = \frac{1}{2}\left(\frac{\partial^2 \phi}{\partial z^2}\right)_0 \int z^2 \rho dv + \frac{1}{2}\left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}\right) \int \frac{x^2 + y^2}{2} \rho dv \ldots\ldots\ldots\ldots\ldots\ldots\ldots \longrightarrow 1.27$$

From Laplace's equation, we have $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$

By substituting this and equation of sphere $r^2 = x^2 + y^2 + z^2$, we get

$$\Delta U_2 = \frac{1}{4}\left(\frac{\partial^2\phi}{\partial z^2}\right)_0 \int \left(3z^2 - r^2\right)\rho dv$$

$$= \frac{1}{4} eQ\left(\frac{\partial^2\phi}{\partial z^2}\right)^0 \longrightarrow 1.28$$

where the quadrupole moment Q is defined as

$$Q = 1/e\int(3z^2 - r^2)\,\rho dv \text{--------}(1.29)$$

This relation shows that

1. $Q = 0$ for a spherically symmetric charge distribution $\{\langle x^2\rangle = \langle y^2\rangle = \langle z^2\rangle = \frac{1}{3}r^2\}$
2. Q is +ve when $3z^2 > r^2$ and the charge distribution is stretched in the z-direction (prolate)
3. In an oblate distribution when $3z^2 < r^2$ and Q is –ve. Since the expression is divided by the electric charge, the dimension of the quadrupole moment is that of an area. As it is very small , hence it is measured in 'barns' in nuclear physics.

(1 barn = 10^{-28} m^2)

1.10 .Summary.

The methods of measuring nuclear radius are divided in to two main categories . One group of method is based on the study of the range of nuclear forces in which the nucleus is probed by a nucleon of light nucleus. Other group of methods studies the electric field and charge distributions of the nucleus in which the nucleus is probed by electron or muon(μ -meson). Prout suggested that all atomic weights were whole numbers and hence could be assumed as the integral multiple of the atomic weight of hydrogen.The slight change in the whole number was explained by the presence of two or more isotopes. The experimental discovery of the neutron led Heisenberg in 1932 to suggest that the nuclei might be composed of proton and neutrons rather than of protons and electrons. Thus for an atom $_zX^A$,the nucleus contains Z protons and (A-Z) neutrons.

For a nuclide $_zX^A$, we write $_ZM^A$ for its actual atomic mass and ZM_H *and* NM_n for the total mass of its constituents . The difference may be written as

$$\{ZM_H + (A-Z)M_n\} -_Z M^A = \Delta_Z M^A$$

This difference $\Delta_Z M^A$ is known as mass defect.

we can say that mass defect $\Delta_z M^A$ appears as an equivalent amount of energy ΔE on forming a nucleus. ΔE is the energy released, due to the decrease of mass, when nucleus is formed by fusing together the requisite number of nucleons; alternately it is the energy required to separate the nucleons of nucleus. It is referred as the binding energy of the nucleus, B.E.

Keywords:
Packing fraction, Mass defect, dipole moment, point charge, Quadrupole moment

Self assessment questions:

1. Define massdefect. Explain the binding energy of nuclei. Analyse the important features of binding energy per nucleon.
2. Derive an expression for magnetic dipole moment of nuclei.
3. Obtain an expression for electric quadrupole moment of nuclei.

Text books

1. Nuclear physics by D.C.Tayal , Himalaya publishing company,Bombay.
2. Nuclear physics by R.C.Sharma, K.Nath&co, Merut
3. Nuclear physics by S.B.Patel.

Unit 1

Lesson 2

NUCLEAR FORCES

The objectives of the lesson are to explain the following:

2.1 Introduction

2.2 Properties of nuclear forces

2.3 Inferences drawn from the experimental data of Deuteron

2.4 Ground state of Deuteron

2.5 Summary

2.1 Introduction:

Earlier we considered the curve of binding energy per nucleon against the mass number (B/A against A curve). The value of B/A is approximately constant and it is about 8Mev/nucleon. This is about a million time higher than the binding energy of an electron in the hydrogen atom (which is 13.6eV). In other words, the force that keeps the nucleus together is much stronger than the electrical force which keeps the atom together. Also , this nuclear force must be stronger than the electrical force between the protons since protons are bound in the nuclei.

What holds the nucleus together? Bound systems are held together by forces. The earth is bound to the sun because of the gravitational force between them. The electron in a hydrogen atom is bound to the nucleus (which is just a proton) because of the account for the existence of nuclei. The gravitational attractive force between two protons is smaller by a factor of about 10^{39} than t he electrical repulsive force between them. In fact, it is customary to neglect the effects of gravity when one consider atoms or nuclei. Thus, one must recognize the existence of a third force in nature, the nuclear force. Further this nuclear force must be much stronger than the electric force; otherwise protons cannot live in close proximity as they do inside a nucleus. This is also expected from the B/A against A curve. When we discussed the size of the nucleus we saw that in a Rutherford α-scattering experiment, a departure from Coulomb law was observed with light elements as targets, only with the most energetic α-particles. This clearly showed that the effect or range of the nuclear force must be of the order of the nuclear radius. In other words the nuclear force is a short-range force. It falls off more rapidly with distance than $1/r^2$.

In general the force between nucleons of a nucleus are of four types.

1. Coulomb's electrostatic forces
2. Nuclear forces.
3. Tensor forces
4. Hard core repulsive forces

2.2. Properties of Nuclear forces:

1. The nuclear forces are short range forces
2. Nuclear forces are saturated forces
3. Nuclear forces are charge independent
4. Nuclear forces are spin dependent

2.2.1.Very short range forces:-

Nuclear forces are very short range of 10^{-15}m. The short range property has been defined from the deuteron problem. At greater distance these forces are negligible.

2.2.2.Saturation Property:-

Nuclear forces are saturated forces. It means that each nucleons attracts only those nucleons which are immediate neighbours. It does not interact with all other nucleons.

2.2.3.Charge independent :-

We know that the mass number 'A' is approximately equal to twice the atomic number 'Z' for the light and intermediate nuclei. It shows that the light nuclei prefer to odd nucleons in N –P pairs i.e. there is a strong interaction between neutron and protons.

For example $_2He^4$ has a more stability, this has mass number A = 4 there are 6 bonds , among these six bonds 4 associated with (n –p) forces and one each with (p –p) & (n –n) forces.

In spectral representation

(a) 1 s singlet configuration 1(np), 1(pp), 1(nn).

(b) 3 s triplet configuration 3(np)

the evidence of the light nuclei is that the corresponding nuclear forces are attractive in each of these states. The neutron excess in heavy nuclei, which is required to balance the coulomb repulsion of the protons, confirms that the 1(nn) (force) is attractive but it is insufficiently attractive to lead a stable di neutron.

The attractive nature of (n-p) (p-p) &(n-n) forces can be easily proved by considering a few simple nuclei. The relative stability of the deutron shows that (n –p) forces is attractive in nature and has appreciable magnitude. The addition of an extra neutron to form T^3 nucleus or of an extra proton to yield $_2He^3$ nucleus is accompanied by a marked increasing of binding energy, partly due to (n-n) and (p-p) forces respectively. In T^3 there a re two (n-p) & one (n-n) forces, while in $_2He^3$ there exist two (n-p) and one (p-p) forces. Their interaction can be represented as

$$T^3 \longrightarrow 3(np) + 1(np) + 1(nn)$$

$$He^3 \longrightarrow 3(np) + 1(np) + 1(pp)$$

The binding energies of T^3 and He^3 are 8.48 Mev and 7.72Mev, respectively. The binding energy difference is 0.76Mev.If (n-n) & (p-p) forces are identical them this difference of energy should be attributed to coulomb energy difference which is

$$E_c = \frac{3}{5}\frac{e^2}{4\pi\varepsilon_0 R^2}\left(Z(Z-1)\right)$$

And r = 2f this gives 0.76Mev, confirming there by (n-n) and (p-p) forces are charge symmetric.

2.2.4.Spin dependent of Nuclear forces:

The coherent scattering is obtained when slow energy neutrons strike the molecular hydrogen. It is known that molecular hydrogen at room temperature consists of 3 parts of ortho hydrogen (two proton with parallel spin) and one part of para hydrogen (spin are antiparallel) and the conversion rate of one into the other is very slow in the absence of a suitable catalyst. The molecular study of hydrogen reveals that ortho –hydrogen ever (0,2, 4 …..) rotational states, indicating there by that para-hydrogen possesses the minimum energy (spin is also zero). Thus, at low temperatures ($\approx 20^0$K) in presence of charcoal (catalyst) almost all hydrogen is in para –form. At low temperature de Broglie wavelength of incident neutron is 7 x 10^{-8} cm which is about nine times the separation of two protons (0.74 x10^{-8} cm) in hydrogen molecule.

Under these conditions (two protons may be assumed at the same place from the point of view of incident and scattered waves) the scattered waves from two protons will interfere appreciably and coherent scattering will take place. The scattering of unpolarised neutrons from para – hydrogen will be a coherent mixture of singlet and triplet scattering for in para-hydrogen spins are antiparallel and hence one of the proton spin will be parallel to and other one antiparallel to the spin of incident neutron. On the other hand the scattering from orthohydrogen either will be singlet or triplet one for the spins of incident neutron. On the other hand the scattering from orthohydrogen either will be singlet or triplet one for the spins of two protons are parallel.

The first theoretical investigation of coherent scattering has been provided by schrodinger and Teller.

The total nuclear spin of neutron and proton is writ ten as.

$$\hat{S} = \hat{S}_n + \hat{S}_p \ldots\ldots\ldots\ldots\ldots\ldots \rightarrow 2.1$$

In the operator form we write

$$\hat{S} . \hat{S} = (\hat{S}n + \hat{S}_p) \ . (\hat{S}n + \hat{S}_p) \ldots\ldots\ldots\ldots \rightarrow 2.2$$

$$\hat{S} . \hat{S} = \hat{S}n . \hat{S}n + \hat{S}n . \hat{S}p + \hat{S}p . \hat{S}n + \hat{S}p . \hat{S}_P$$

$$\hat{S} . \hat{S} = \hat{S}n . \hat{S}n + 2 \hat{S}n . \hat{S}p + \hat{S}p . \hat{S}p$$

$$2 \hat{S}n . \hat{S}p = \hat{S} . \hat{S} - \hat{S}n . \hat{S}n - \hat{S}p . \hat{S}p$$

$$\hat{S}n . \hat{S}p = \tfrac{1}{2} \{ \hat{S} . \hat{S} - \hat{S}n . \hat{S}n - \hat{S}p . \hat{S}p\}$$

$$\hat{S}n \cdot \hat{S}p = \tfrac{1}{2} \{ S(S+1) - S_n(S_n+1) - S_p(S_p+1) \} \ldots\ldots\ldots\ldots \rightarrow 2.3$$

As $S_n = Sp = \tfrac{1}{2}$.; Neutron and proton are ½ spin particles.

Equation (2.3) becomes

$$\hat{S}n \cdot \hat{S}p = \tfrac{1}{2}\ S(S+1) - 3/4 \ldots\ldots\ldots\ldots \rightarrow 2.4$$

Introducing now Pauli spin oplerator $\hat{S} = \frac{1}{2}\sigma\hat{}\ldots\ldots\ldots \rightarrow 2.5$

We write $\sigma_n\hat{} \cdot \sigma_p\hat{} = 4\,\hat{S}n \cdot \hat{S}p$

$$= 2\ S(S+1) - 3 \ldots\ldots\ldots\ldots \rightarrow 2.6$$

$$\sigma_n\hat{} \cdot \sigma_p\hat{} = -3 \text{ for singlet state ; } S = 0 \ldots\ldots\ldots\ldots \rightarrow 2.7$$

$$\sigma_n\hat{} \cdot \sigma_p\hat{} = 1 \text{ for triplet state; } S = 1 \ldots\ldots\ldots\ldots \rightarrow 2.8$$

with these conventions, a single expression can represent scattering, lengths for singlet state and triplet state i.e.

$$a_{t,s} = \tfrac{1}{4}(3a_t + a_s) + \tfrac{1}{4}(a_t - a_s)\,\sigma_n\hat{} \cdot \sigma_p\hat{} \ldots\ldots\ldots\ldots \rightarrow 2.9$$

To calculate the amplitude of the scattered of the scattered wave f_{Hy} the amplitude should be added directly

$$-f_{H_Z} = a^{1}_{t,s} + a^{11}_{t,s} = \tfrac{1}{2}(3a_t + a_s) + \tfrac{1}{4}(a_t - a_s)\,\sigma_n\hat{}\,(\sigma_{p1}\hat{} + \sigma p2\hat{})$$

$$= 1/2\,(3a_t + a_s) + (a_t - a_s)\,\hat{S}n \cdot \hat{S}_H \ldots\ldots\ldots\ldots \rightarrow 2.10$$

where $\sigma_{p1}\hat{}$ & $\sigma p2\hat{}$refers to the two protons and

$(\sigma_{p1}\hat{} + \sigma p2\hat{}) = 2\,\hat{S}_{H.}$ total spin operator of hydrogen molecule.

Now the scattering cross-section are given by

$\sigma_{para} = 4\Pi\,(-f_{H_Z})p2 = \Pi\,(3a_t + a_s)^2$ ………………→2.11

and $\sigma_{ortho} = 4\Pi\,\{\tfrac{1}{2}(3a_t + a_s) + (a_t - a_s)\,\hat{S}_n.\hat{S}_H\}^2$ …………..→2.12

$\hat{S}_n.\hat{S}_H = 4\Pi\{\tfrac{1}{4}(3a_t + a_s)^2 + (3a_t + a_s)(a_t - a_s)\,\hat{S}_n.\hat{S}_H + (a_t - a_s)^2\,\hat{S}_n^2.\hat{S}_H^2\}$ …….→2.13

The terms in equation (2.13) should be averaged for all originations of spin of neutrons and for this we write

$$<\{\hat{S}_n.\hat{S}_H\}^2> = <\{\hat{S}_{nx}\hat{S}_{Hx} + \hat{S}_{ny}\hat{S}_{Hy} + \hat{S}_{nz}\hat{S}_{Hz}\}^2>$$

$$= <\hat{S}_{nx}^2\hat{S}_{Hx}^2 + \hat{S}_{ny}^2\hat{S}_{Hy}^2 + \hat{S}_{nz}^2\hat{S}_{Hz}^2 + 2\hat{S}_{nx}\hat{S}_{Hx}\hat{S}_{ny}\hat{S}_{Hy} + 2\hat{S}_{ny}\hat{S}_{Hy}\hat{S}_{nz}\hat{S}_{Hz} + 2\hat{S}_{nx}\hat{S}_{Hx}\hat{S}_{nz}\hat{S}_{Hz}> \text{ ……………}\rightarrow 2.14$$

This gives $<\{\hat{S}_n.\hat{S}_H\}> = 0$

$$<\{\hat{S}_n.\hat{S}_H\}^2> = <\hat{S}_{nx}^2\hat{S}_{Hx}^2 + \hat{S}_{ny}^2\hat{S}_{Hy}^2 + \hat{S}_{nz}^2\hat{S}_{Hz}^2>$$

$$\hat{S}_{nx}^2 = \hat{S}_{ny}^2 = \hat{S}_{nz}^2 = \tfrac{1}{4}$$
$$\hat{S}_{Hx}^2 + \hat{S}_{Hy}^2 + \hat{S}_{Hz}^2 = \hat{S}_H^2$$
$$<\{\hat{S}_n.\hat{S}_H\}^2> = \tfrac{1}{4}\hat{S}_H^2 = \tfrac{1}{4}S_H(S_H + 1)$$
$$= 1/4\;1(1+1) = \tfrac{1}{2}$$

For expression for cross-section can now be written as

$$\sigma_{para} = \Pi\,(3a_t + a_s)^2$$

$$\sigma_{ortho} = 4\Pi\,\{\tfrac{1}{4}(3a_t + a_s)^2 + 1/2\,(a_t - a_s)^2\}$$

$$\sigma_{ortho} = \sigma_{para} + 2\Pi(a_t - a_s)^2 \text{ ………………}\rightarrow 2.16$$

If n-p forces is spin – independent, $a_t = a_s$ and $\sigma_{ortho} = \sigma_{para}$. But the experim ental values.

σ_{para} = 4 barns & σ_{ortho} = 125 barns.

The large difference between two cross-section clearly suggests that n-p force is spin-dependent..

2.3 Inferences drawn from the experimental data of Deuteron:

The deuteron is the only two nucleon bound system and consists of one neutron and one proton. The experimentally measured data together with the inference drawn from them are outlined below.

1. charge = +e
2. Mass = 2.014735 amu
3. Radius of the deuteron: the radius of deuteron has been measured with the help of high energy electron scattering experiment. 2.1Fermi(1Fermi = 10^{-15} m).
4. Binding Energy: The binding energy of the deuteron is measured with high accuracy either by photo disintegration of deuteron or by (n-p) capture. The value is (2.225±0.002)Mev.
5. Angular Momentum(spin) : $1\hbar$
6. The result implies that in the deuteron ground state the intrinsic spin of neutron and proton have parallel alignment and no contribution from orbital motion of the two nucleons results. It further mean that in the ground state two nucleons lie in L = 0 i.e in spherically symmetric state.
7. Statistics: Bose – Einstein
8. Parity: Even
9. The magnetic moment:

It is measured with the help of magnetic resonance absorption method in which quantum energy needed to flip over a nucleus with its magnetic moment in the magnetic field is measured. Its value is 0.8754nm. The magnetic moment of deuteron is the addition of magnetic moments of proton and neutron but not difference of

μ_p =(2.7925±0.0001)nm μ_n = (- 1.9128 ±0.0001)nm

$$\mu_d = \mu_p + \mu_n = 0.8797\text{nm}$$

using the result of point (5), we say that deuteron is in the ground state and is spherically symmetric(L=0) and hence the magnetic moment of deuteron is due to the addition of proton and neutron magnetic moments and it cannot be due to the subtraction of the two. However, μ_d is not equal to the exact sum of μ_p and μ_n and difference (about 3%) is well outside the experimental error. But since the difference is small, we can infer the following points on the basis of above results.

(a) The ground state of deuteron is a triplet ($3s_1$) state i.e the spins of two nucleons are parallel.

(b) The orbital angular momentum in the ground state is zero. Because the other combination (L = 1, S =1) and L= 2 ,S =1) give the value of μ_d as0.677 nm and 0.323 nm which are very different from the experimental value.

(c) The neutron spin is ½

(d) Quadrupole moment: The quadrupole moment of deuteron has been measured by Rabi and his co-workers using radio frequency techniques and the result gives Q_d = 0.00202 barns (1 barn = 10^{-24} cm^2).It implies that charge distribution in the ground state is not spherically symmetric because a spherical charge distribution leads a value for $Q_d = 0$ or for the $\frac{z^2}{r^2} = \frac{1}{3}$.The result also indicate that charge distribution is of prolate shape,i.e., elongated along the z axis. This in turn emphasizes that spins of the particles are more found lined up one after the other rather than side by side.

Now the problem is how to account for the discrepancy in magnetic moment (of the order of 3%) and a non-zero quadrupole moment. The explanation was furnished by Schwinger and Rarita who suggested that ground state wave function is not strictly spherically symmetric(no strictly spherical charge distribution) but it is the combination of one spherically symmetric function and one non-spherically symmetric function.

The existence of spherically symmetric state 3_{S_1} is proved reasonable due to the very close resemblance of the μ_d with the sum $(\mu_n + \mu_p)$. The possible asymmetric states which are consistent with observed deuteron total angular momentum of unity, I =L+S = 1, three possible values of L which satisfy this relation are $^3P_1$, $^{1}P_{1,}$ $^3D_1$. The two P states, however, give opposite parity for the system and these are excluded on the ground of conservation of parity which does not permit a bound system to have sometime the even parity and sometime the odd parity. Thus, the only alternative left is that ground state wave function is combination of $^3S_1$ and $^3D_1$ states. The function may be written as

$$\psi = a_s\psi_s + a_D\psi_D \text{......................} 2.17$$

where ψ_s *and* ψ_D are the normalized wave functions in S and D states with total angular momentum I = 1 and ψ is normalized by writing

$$a_s^2 + a_D^2 = 1 \text{..................} 2.18$$

Now if it is assumed that deuteron spends part of its time is D-state then it will give some orbital magnetic moment which in turn explains the lack additivity of deuteron magnetic moment. The $^3D_1$ state also gives a charge distribution which is not spherically symmetric and explain the non-zero quadrupole moment. The observed values of magnetic moment and quadrupole moment are such that they demand 3.9% probability of existence of deuteron in $^3D_1$ and 96.1% in $^3S_1$ states. The probability in

${}^3D_1$ state is given by

$$\frac{a_D^2}{a_s^2 + a_D^2} = 0.039$$
$$= 3.9\%$$

Alternately, the above discussion may be summarized by saying that deuteron spends 96.1% time in ${}^3S_1$ and 3.9% time in ${}^3D_1$ and therefore, ground state of deuteron is mixture of ${}^3S_1$ and ${}^3D_1$ states.

GROUND STATE OF DEUTERON:

It has been said that deuteron is the simplest of two nucleon system and consists of one neutron and one proton. It is also mentioned that in most of the time deuteron is in the spherically symmetric state. Therefore, the following assumptions are

i) Deuteron consists of two particles of roughly equal ($M_n \cong M_p = M$) mass M., making there by the reduced mass

$$\mu = \frac{MM}{M+M} = \frac{M}{2} \quad \text{.......................2.19}$$

ii) The force between the two particles is short range and attractive and acts along the line joining the two particles and does not depend upon the orientations. It is a central force. However, this assumption appears to be in correct because, it can not account for the quadrupole moment of deuteron. But it is assumed here for the sake of mathematical simplicity.

iii)Since central force is conservative force and can be written as the gradient of some potential V(r)

$$\nabla V(r) = gradV(r)$$
$$\hat{F} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)V(x,y,z)$$
$$\hat{F} = \hat{i}F_x + \hat{j}F_y + \hat{k}F_z$$

The Schrodinger equation for deuteron in the centre of mass system may be written as

$$\nabla^2\psi + \frac{2\mu}{\hbar^2}(E-V)\psi = 0 \longrightarrow 2.20$$

where μ is reduced mass

∇^2 is laplacian operator $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

V is the potential energy describe the force between two nucleons.

E is the total energy of the system which is equal to B.E. of the deuteron.

Ψ is wave function.

In terms of spherical polar coordinates the schrodinger equation (2.20) is translated.

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2 Sin\theta}\frac{\partial}{\partial\theta}\left(Sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2 Sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2} + \frac{2\mu}{\hbar^2}(E-V)\psi = 0 \longrightarrow 2.21$$

In the central force assumptions the wave function $\Psi(r,\theta,\Phi)$ is expressed as

$$\psi(r,\theta,\phi) = U_l^{\,1}(r)Y_{l,m}(\theta,\phi) \longrightarrow 2.22$$

substitute (2.22) in (2.21)

Multiplying through out by r^2 and then write radial and angular part terms separately

$$\frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\left(U_l^1(r)Y_{l,m}(\theta,\phi)\right)\right)\right]+\frac{1}{r^2 Sin\theta}\left[\frac{\partial}{\partial\theta}\left(Sin\theta\frac{\partial}{\partial\theta}\left(U_l^1(r)Y_{l,m}(\theta,\phi)\right)\right)\right]+$$

$$\frac{1}{r^2 Sin^2\theta}\left[\frac{\partial^2}{\partial\phi^2}\left(U_l^1(r)Y_{l,m}(\theta,\phi)\right)\right]+\frac{2\mu}{\hbar^2}(E-V)\left(U_l^1(r)Y_{l,m}(\theta,\phi)\right)=0$$

$$\Rightarrow\frac{1}{U_l^1(r)}\left[\frac{d}{dr}\left(r^2\frac{\partial U_l^1(r)}{\partial r}\right)+\frac{2\mu r^2}{\hbar^2}(E-V)\left(U_l^1(r)\right)\right]$$

$$=-\frac{1}{Y_{l,m}(\theta,\phi)}\left[\frac{1}{Sin\theta}\left[\frac{\partial}{\partial\theta}\left(Sin\theta\frac{\partial}{\partial\theta}Y_{l,m}(\theta,\phi)\right)\right]+\frac{1}{Sin^2\theta}\frac{\partial^2}{\partial\phi^2}\left(Y_{l,m}(\theta,\phi)\right)\right]\longrightarrow 2.23$$

Now equate the radial part term to some constant $l(l+1)$, *we have*

$$\Rightarrow\frac{1}{U_l^1(r)}\left[\frac{d}{dr}\left(r^2\frac{dU_l^1(r)}{dr}\right)+\frac{2\mu r^2}{\hbar^2}(E-V)\left(U_l^1(r)\right)\right]=l(l+1)\longrightarrow 2.24$$

multiplying eqn(5) bothsides by $\frac{U_l^1(r)}{r^2}$

$$\Rightarrow\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dU_l^1(r)}{dr}\right)+\frac{2\mu}{\hbar^2}(E-V)\left(U_l^1(r)\right)=l(l+1)\frac{U_l^1(r)}{r^2}\longrightarrow 2.25$$

$$\Rightarrow\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dU_l^1(r)}{dr}\right)+\frac{2\mu}{\hbar^2}\left[E-V(r)-\frac{l(l+1)\hbar^2}{2\mu r^2}\right]U_l^1(r)=0\longrightarrow 2.26$$

It is clear that the last term in the bracket, i.e., $\frac{l(l+1)\hbar^2}{2\mu r^2}$ is an addition to the actual potential V(r) and is known as centrifugal force, because its derivative with respect to 'r' is equal to classical centrifugal force when angular momentum is $\sqrt{l(l+1)}\hbar$. Thus , l represents orbital angular momentum quantum number. The function of centrifugal force is to disrupt the nucleus and therefore, the potential V(r) compensates the effect of centrifugal force at least over certain range of 'r'. The effect of centrifugal force will be minimum in the ground state (l=0) and the V(r) will be fully utilized in the binding the nucleons together.

In the groundstate $U_l^1(r)$ is spherically symmetric and we can write

$$U_l^1(r) = \frac{U(r)}{r} \longrightarrow 2.27$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d}{dr}\left(\frac{U(r)}{r}\right)\right) + \frac{2\mu}{\hbar^2}[E - V(r)]\frac{U(r)}{r} = 0$$

$$\Rightarrow \frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{1}{r}\frac{dU(r)}{dr} - U(r)\right) + \frac{2\mu}{\hbar^2}[E - V(r)]\frac{U(r)}{r} = 0$$

$$\Rightarrow \frac{d^2U(r)}{dr^2} + \frac{2\mu}{\hbar^2}[E - V(r)]U(r) = 0$$

$$\frac{d^2U}{dr^2} + \frac{2\mu}{\hbar^2}[E - V(r)]U(r) = 0 \longrightarrow 2.28$$

Since force is every where attractive, the V(r) is negative and decreases with decreasing value of 'r'. The following shapes have been proposed for potential V(r)

1. Exponential potential $V(r) = -V_0 e^{-\frac{r}{b}}$
2. The Gausian potential $V(r) = -V_0 e^{-\frac{r^2}{b^2}}$
3. Yukawa Potential $V(r) = -V_0 \frac{e^{-\frac{r}{b}}}{r/b}$
4. Potential with hard repulsive core

$V(r) = -V_0\, e^{-r/b}$ for $r > b^1$

$V(r) = \alpha$ for $r < b^1 < b$

5.Square well potential $V(r) = 0$ for $0 < r < b^1$ Region (1)

$V(r) = -V_0$ for $b^1 < r < b$ Region (2)

$V(r) = 0$ for $r > b+b^1$ Region (3)

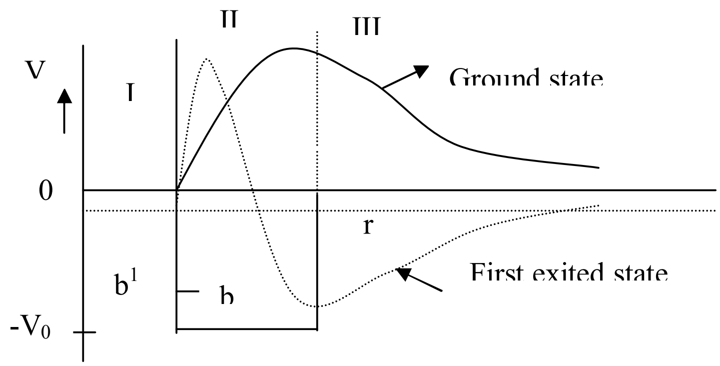

Fig: 2.1 The shapes of V(r) and u(r)

In all above representation 'b' is the range of the potential V(r) and V_0 is the depth of the potential well. The existence of hard repulsive core prevents the particles to come more closer than the distance b^1 and its presence is verified by scattering studies. Since the wave function is zero in the region I, we have only to solve it for the region II and III. The wave equation takes the following forms in region II and III

$$\frac{d^2U}{dr^2}+\frac{M}{\hbar^2}(-B+V_0)U(r)=0 \quad \text{……… Region (2)…………….2.29}$$

$$\frac{d^2U}{dr^2}+\frac{M}{\hbar^2}(-B)U(r)=0 \quad \text{……………….. Region (3)…………..2.30}$$

Where 'B' is Binding energy of deuteron.

In eqn (2.29) & (2.30) V_0 & B are +ve numbers and B value is 2.225Mev

Put $\frac{M}{\hbar^2}(V_0-B)=k^2$ in eqn (2.29)

$\frac{M}{\hbar^2}B=\alpha^2$ in equation (2.30)

$$\frac{d^2U}{dr^2} + k^2U = 0 \longrightarrow 2.31$$

$$\frac{d^2U}{dr^2} - \alpha^2U = 0 \longrightarrow 2.32$$

General solution of eqn (2.31) & (2.32) are

U(r) = A Sin kr + B Cos kr $\longrightarrow$ 2.34

$$U(r) = ce^{-\alpha r} + De^{\alpha r} \longrightarrow 2.35$$

The following boundary condition must be imposed

By using the boundary conditions

For well behaved function $\psi(r) = \frac{u(r)}{r}$ $as \;\; r \to 0, the \; \psi \;\; is \;\; finite \;\; and \;\; r \to \infty, \; \psi \;\; vanishes$

$$\psi(r) = \frac{u(r)}{r} = Ak \;\; \underset{r\to 0}{Lt} \frac{Sinkr}{kr} + B\frac{Coskr}{r}$$

As $r \to 0 \;\; finite \; = finite + B \;\; Infinite(If \;\; there \;\; is \;\; no \;\; restriction \;\; on \;\; B$

In order to make LHS =RHS, B should be zero.

For well behaved function the solutions of eqn (2.34) becomes

$$\Rightarrow U(r) = ASinkr \longrightarrow 2.36$$

For well behaved function the solution (2.35) also becomes

$$\Rightarrow U(r) = ce^{-\alpha r} \longrightarrow 2.37$$

Further the wave function U(r) is also zero at r = b^1

There fore, the solution of equation can be written as

U(r) = Sin (r – b^1)

At boundary of the two regions i.e. r = b + b^1

The function u(r) and its derivative are continuous i.e., at r = b + b^1

This gives ASinkb $= ce^{-\alpha\left(b+b^1\right)}$

$$ASinkb = ce^{-\alpha\left(b+b^1\right)} \longrightarrow (2.38)$$

$$AkCoskb = -c\alpha c^{-\alpha\left(b+b^1\right)} \longrightarrow (2.39)$$

Divide (2.39) by (2.38)

$$KCotkb = -\alpha \longrightarrow (2.40)$$

$$\Rightarrow Cotkb = -\frac{\alpha}{k}$$

From eqn (2.29) & (2.30) $K = \dfrac{\sqrt{m(V_0 - B)}}{\hbar}, \alpha = \dfrac{\sqrt{mB}}{\hbar}$

$$Cotkb = -\frac{\alpha}{k} = \frac{\sqrt{mB}}{\hbar} \Bigg/ \frac{\sqrt{m(V_0 - B)}}{\hbar} = -\sqrt{\frac{B}{V_0 - B}} \longrightarrow 2.41$$

As B is much smaller than V_0 it implies that Cotkb is negative and small, which means that kb should be slightly larger $(2n+1)\frac{\pi}{2}$ where n = 0,1,2,3,…………..

$$kb = \frac{\pi}{2}$$

$$k^2 b^2 \approx \frac{\pi^4}{4} \longrightarrow (2.42)$$

Substitute 'k' value in eqn (2.42)

$$\Rightarrow \frac{m(V_0 - B)b^2}{\hbar^2} = \frac{\pi^2}{4}$$

neglecting B,

$$\Rightarrow \frac{m(V_0)b^2}{\hbar^2} = \frac{\pi^2}{4} \Rightarrow V_0 = \frac{\pi^2 \hbar^2}{4mb^2}$$

$$V_0 = \frac{\pi^2 \hbar^2}{16\pi^2 mb^2} = \frac{h^2}{16b^2 m}$$

If we take range b = 2 fermi, then

$$= \frac{\left(6.67\times10^{-34}\right)^2}{16\left(2\times10^{-15}\right)^2\left(1.674\times10^{-27}\right)\left(1.6\times10^{-15}\right)}$$
$$= 25.5Mev$$

In the figure(2.1), the plot of U(r) versus 'r' shows that beyond r = b, the function U(r) decreases exponentially with 'r'. The radial distance at which the amplitude of function $U(r) = ce^{-\alpha r}$ is 1/e of its maximum amplitude is often called the radius of the deuteron.

$$\text{Radius} \qquad R_d = \frac{1}{\alpha} = \frac{\hbar}{\sqrt{mB}} = 4.31\times10^{-15}$$

The ground state total wave function in the two regions can be written as

$$\psi(r,\theta,\phi) = U_l^{\,1}(r)Y_{l,m}(\theta,\phi)$$
$$U(r) = ASink\left(r - b^1\right)$$
$$U(r) = ce^{-\alpha r}$$
$$U_l^{\,1}(r) = \frac{U(r)}{r}$$

$$\psi_{II} = \frac{U(r)}{r}Y_{0,0} = \frac{1}{\sqrt{4\pi}}\frac{U(r)}{r} = \frac{1}{\sqrt{4\pi}}\frac{ASink(r-b^1)}{r} \text{ .... Region (2)}$$

$$\psi_{III} = \frac{1}{\sqrt{4\pi}}\frac{ce^{-\alpha r}}{r} \text{ .... Region (3)}$$

2.5.Summary:

The value of B/A is approximately constant and it is about 8Mev/nucleon. This is about a million time higher than the binding energy of an electron in the hydrogen atom (which is 13.6eV). In other words, the force that keeps the nucleus together is much stronger than the electrical force which keeps the atom together. Also , this nuclear force must be stronger than the electrical force between the protons since protons are bound in the nuclei.

This is also expected from the B/A against A curve. When we discussed the size of the nucleus we saw that in a Rutherford α-scattering experiment, a departure from Coulomb law was observed with light elements as targets, only with the most energetic α- particles. This clearly showed that the effect or range of the nuclear force must be of the order of the nuclear radius. In other words the nuclear force is a short-range force. It falls off more rapidly with distance than $1/r^2$.

The nuclear forces are short range forces, saturated forces, charge independent and spin dependent.

The observed values of magnetic moment and quadrupole moment are such that they demand 3.9% probability of existence of deuteron in ${}^3D_1$ and 96.1% in ${}^3S_1$ states.

Keywords:
Nuclear forces, Deuteron , Spin

Self assessment questions:

1. What are the characteristics of nuclear forces?
2. How do you determine the binding energy, magnetic moment and quadrupole moment of deuteron?
3. Obtain an expression for the total wave function for the ground state of deuteron on the basis of central force assumption..

Text books

1. Nuclear physics by D.C.Tayal , Himalaya publishing company,Bombay.
2. Nuclear physics by R.C.Sharma, K.Nath&co, Merut
3. Nuclear physics by S.B.Patel

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Unit 1

Lesson 3

NUCLEON –NUCLEON SCATTERING

The objectives of the lesson are to explain the following:

3.1.Introduction: Since we use the principles of wave mechanics on nuclear problems and then by comparison with experimental data, find a consistent description of the nuclear forces acting between two nucleons (two body problem). We have seen the study of deuteron (only bound state of two nucleons) in previous chapter. There are two general methods of investigation, the study of n-p and p-p scattering events over a wide range of energy.

3.2.Proton – Proton scattering at low Energies:

The P-P scattering is the only way to get direct evidence on proton-proton force. There are two essential differences between p-p and n-n scattering. First, the p-p scattering is caused not only by nuclear force but also by the coulomb force. Second, the scattering and scattered particles are identical and obey the pauli exclusion principle and therefore, the wave function describing the two protons must change sign on the interchange of the two particles Hence a symmetric space wave function (s, d states etc) can only be associated with an anti-symmetric spin wave function. Where as a symmetric (triplet) spin wave function is required for an anti-symmetric space wave function.

Experimentally the study of p-p scattering, is capable much higher accuracy than n-p scattering, for the following reasons.

1. Protons are easily available over a wide range of energies
2. Protons can be made mono energetic
3. protons can be produced in well collimated beam.
4. Protons can be easily detected by their ionizing properties.
5. Protons undergo coulomb scattering. This increases the sensitivity in case one of the scattering probabilities is small and also gives sign of the phase shifts resulting from nuclear scattering
6. The proton combination obeys Fermi statistics. This simplifies the analysis of p-p scattering.

We are now interested in obtaining a theoretical expression for the different elastic scattering cross-section of protons by protons. The theory is more complicated than that of n-p scattering because the coulomb potential appreciably distorts the incident wave even at finite distance.

The radial wave equation for the proton-proton system is

$$\frac{d^2U}{dr^2}+\frac{2m}{\hbar^2}\left[E-V(r)-\frac{e^2}{4\pi\varepsilon_0 r}-\frac{l(l+1)\hbar^2}{2mr^2}\right]\mu=0\longrightarrow 3.1$$

where V(r) is the square well nuclear potential.

The Rutherford cross-section for coulomb scattering, of particle of charge z_1, e by a particle of charge z_2e the center of mass system is

$$d\sigma=\left[\frac{(z_1e)^2(z_2e)^2}{4(4\pi\varepsilon_0)^2m^2v^4}Co\sec^4\frac{\theta}{2}\right]2\pi Sin\theta d\theta\longrightarrow 3.2$$

where 'v' is the velocity of the incident particle, ' m' is the reduced mass of the system and 'θ' is the angle of scattering in the center of mass system. For P-P scattering $z_1 = z_2 = 1$ m = M/2, 'θ' = 2θ₁ and $Sin\theta\, d\theta = 4Sin\theta_1\; Cos\theta_1\; d\theta$ (where θ_1 is in laboratory system.)

Thus in laboratory system, the cross-section becomes

$$d\sigma = \frac{e^4}{(4\pi\varepsilon_0)^2 E_0^{\,2}}\left[\frac{1}{Sin^4\theta_1} + \frac{1}{Cos^4\theta_1}\right] Cos\theta_1\, 2\pi Sin\theta_1 d\theta_1 \longrightarrow 3.3$$

Where $E_0 = \frac{1}{2}mV^2$ (Kinetic energy of incident proton in the laboratory system). The term containing $Cos^4\theta_1$ is added because each proton at angle θ_1 in the L – system is accompanied by a recoil proton at an angle(90- θ_1)

Eqn (3.3) does not agree with experiment even at low energies where nuclear scattering is negligible. Mott introduced a third term from wave mechanical consideration

Thus eqn (3.3) becomes

$$d\sigma = \frac{e^4}{(4\pi\varepsilon_0)^2 E_0^{\,2}}\left[\frac{1}{Sin^4\theta_1} + \frac{1}{Cos^4\theta_1} - \frac{Cos\left\{\dfrac{e^2}{4\pi\varepsilon_0\hbar v}\log\tan^2\theta_1\right\}}{Sin^2\theta_1 Cos^2\theta_1}\right] Cos\theta_1\, 2\pi Sin\theta_1 d\theta_1 \longrightarrow 3.4$$

The -ve sign is because the proton obey Fermi Statistics. Second and third terms drop out for unlike charged particles. For proton energies, above 1Mev, $\frac{e^2}{4\pi\varepsilon_0\hbar v} \succ \frac{1}{7}$ and cosine term is nearly unity. The third term is called Mott term. For, $\theta_1 = 0 \;\; or \;\; \frac{\pi}{2},\;\; for \;\; E_0 \succ 1Mev,$ the Mott term is simplified and the above equation reduces to

$$d\sigma = \frac{e^4}{(4\pi\varepsilon_0)^2 E_0^{\,2}}\left[\frac{1}{Sin^4\theta_1} + \frac{1}{Cos^4\theta_1} - \frac{1}{Sin^2\theta_1 Cos^2\theta_1}\right] Cos\theta_1\, 2\pi Sin\theta_1 d\theta_1 \longrightarrow 3.5$$

The main difference between proton and neutron seems to be of electric charge and the nuclear force apparently does not arise from charge. Therefore, in P-P scattering at low energies it is expected that only the l= 0 scattering processes will be affected by the nuclear potential, just as in n-p scattering. For s wave, nuclear scattering the complete differential scattering cross-section in laboratory system is given by

$$\frac{d\sigma}{d\Omega}=\frac{e^4}{(4\pi\varepsilon_0)^2 E_0{}^2}\left[\begin{array}{l}\dfrac{1}{Sin^4\theta_1}+\dfrac{1}{Cos^4\theta_1}-\dfrac{1}{Sin^2\theta_1 Cos^2\theta_1}-\dfrac{8\pi\varepsilon_0\hbar v Sin\delta}{e^2}\\ \left\{\dfrac{Cos\left[\delta_0+\dfrac{e^2}{4\pi\varepsilon_0\hbar v}\log Sin^2\dfrac{\theta}{2}\right]}{Sin^2\dfrac{\theta}{2}}+\dfrac{Cos\left[\delta_0+\dfrac{e^2}{4\pi\varepsilon_0\hbar v}\log Cos^2\dfrac{\theta}{2}\right]}{Cos^2\dfrac{\theta}{2}}\right\}\\ +\left(\dfrac{8\pi\varepsilon_0\hbar v}{e^2}\right)^2 Sin^2\delta_0\end{array}\right]Cos\theta_1 \longrightarrow 3.6$$

In this equation 1st , 2nd and 3rd terms are due to coulomb repulsion, the 5th term due to nuclear scattering and 4th due to interference between the coulomb and nuclear scattering effects. This relation reduces to eq(3.5) for a pure coulomb field when nuclear phase shift δ_0 is zero, i.e. when there is no nuclear scattering. The 4th term is useful in determining δ_0 when δ_0 is small , because it is linear in Sin δ_0 and it is large compared to the last term which involves $Sin^2\delta_0$ .The differential cross section is a complicated function of energy E_0, the angle of scattering and the phase shift δ_0. Differential cross section for P-P elastic scattering for 2.4 Mev protons is shown in fig. At small scattering angles the scattering is essentially pure coulomb scattering. The nuclear scattering predominates in the central region of angles, near a c.m angle of 90^0. At forward and backward angles, the dips are caused by interference between nuclear and coulomb scattering.

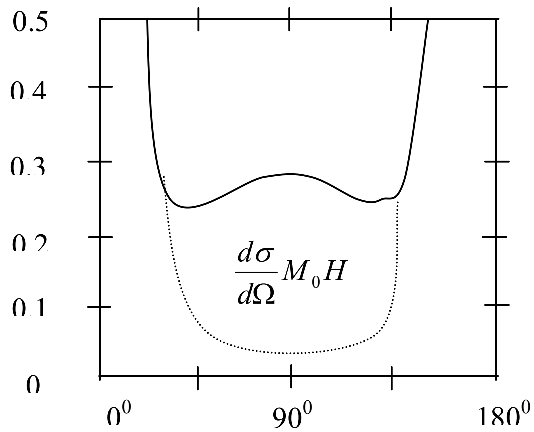


Fig. 3.1 Differential Scattering cross section for P-P scattering.

3.3.NEUTRON –PROTON SCATTERING AT LOW ENERGIES:

The fact that the deuteron is a bound system, shows that attractive forces exist between neutrons and protons. Further information on the inter-nucleon forces a can be obtained from a study of the scattering of free neutrons by protons. In such experiments a parallel bean of neutrons is allowed to impinge upon a target containing hydrogen atom and the number of neutrons deflected through various angles is determined as a function of neutron energy. Since neutrons have no charge, they are unaffected by the electrostatic field and their scattering will directly reflect the operation of the nuclear forces.

Two kinds of the reactions can be involved in neutron proton interaction .One scattering and other radiative capture .The latter has low probability and cross section for high energy neutrons ,as the cross section for the competing radiative capture reaction decreases with (1/V) ,where V is the neutron velocity. In practice protons are bound in molecules. The chemical binding energy of the proton in a molecule is about 0.1eV.Thus for neutron energies >1eV the proton can be assumed as free. This sets a lower limit to the neutron energy . If the neutron energy is less than 10MeV,only the S wave overlaps with the nuclear potential and scattered.

In the center of mass system, the Schrodinger equation for the two body(n-p system) problem is

$$\nabla^2\psi + \frac{M}{\hbar^2}\left(E - V(r)\right)\psi = 0 \longrightarrow 3.7$$

where M = proton or neutron mass = 2 × reduced mass of the system(m).

E= Incident K. E in c.m system

= ½ (incident K.E in L- co-ordinates)

and V(r) = Inter – nucleon potential energy.

At large distance from the center of scattering the solution of this equation is expected to be of the form

$$\psi = e^{ikz} + \frac{e^{ikr}}{r} f(\theta) \longrightarrow 3.8$$

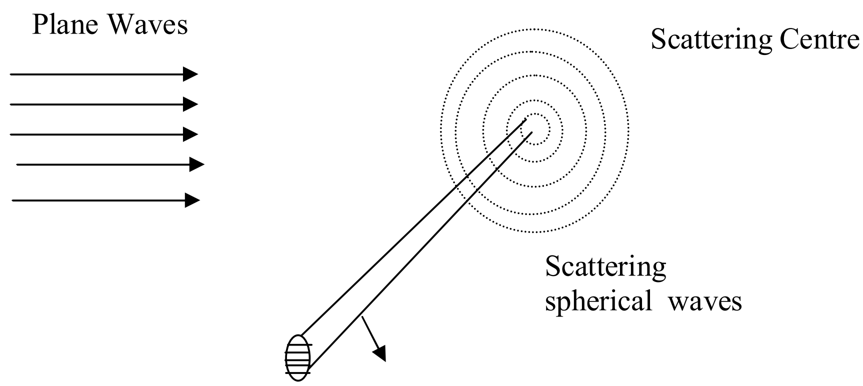


Fig.3.2. Scattering Process

The term represents a plane wave describing a beam of particles moving in the

z-direction towards the origin. The second term represents the scattered wave. The complex quantity f(θ) is the scattering amplitude in the direction θ and is to be evaluated in terms of K. In the case of a spherically symmetric potential the entire arrangement is axially symmetric about the incident direction and hence does not depend on the azimutal angle Φ the 1/r dependence is necessary for the conservation of particles in the outgoing wave. The volume of a spherical shell, between r and r + dr is and hence the density of particles in it or the probability of finding one particle in the spherical shell must vary with $1/r^2$ which is proportional to the square of the amplitude of the scattered wave in the shell. Hence the amplitude of the scattered wave must vary with 1/r.

To compute the differential scattering cross section, we must find the number of particles dN scattered in unit time by one target nucleus in to a solid angle d and the incident flux F. If V is the speed of an incoming particle with respect to the scattered , then

Incoming flux of particles F = $\psi_{in}^{*}\psi_{in} v = v$

Similarly dN is equal to the flux of scattered particle $\psi_{sc}^{*}\psi_{sc} v$ multiplied by the area r^2 dΩ on a spherical surface of radius r and is given by

$$dN = \psi_{sc}^{*}\psi_{sc} v r^2 d\Omega = |f(\theta)|^2 v d\Omega$$

The differential cross section $d\sigma = |f(\theta)|^2 v d\Omega = |f(\theta)|^2 d\Omega$

$$\text{or}\, \sigma = \int |f(\theta)|^2 d\Omega = 2\pi \int |f(\theta)|^2 Sin\theta d\theta \longrightarrow 3.9$$

First of all let us consider the wave equation (3.7) in the absence of a scattering center (v(r) = 0 for all values of r)

$$\nabla^2\psi + \left(\frac{mE}{\hbar^2}\right)\psi = 0 \longrightarrow 3.10$$

This has the solution $\psi = e^{ikz}$

$$\text{Where } \frac{1}{\lambda} = \sqrt{\frac{mE}{\hbar^2}} \longrightarrow 3.11$$

Lord Rayleigh proposed that this type of wave function can be expanded in to a series in terms of spherical harmonic functions Thus equation (3.11) can be written as an infinite series.

$$\psi = e^{ikz} = e^{ikrCos\theta} = \sum_{l=0}^{\alpha} R_l(r) Y_{l,0}(\theta) \longrightarrow 3.12$$

the radial function R_l (r) is the solution of the radial part of equation (3.10)

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)+\left(K^2-\frac{l(l+1)}{r^2}\right)R=0 \longrightarrow 3.13$$

This equation has two solutions, one is not finite at the origin and cannot represent the plane wave. The other is finite at origin and can be represented in terms of spherical Bessel functions as

$$R_l(r)=i^l\sqrt{4\pi(2l+1)}\,j_l(kr) \longrightarrow 3.14$$

Thus square of this gives the r- dependence of the probability density for each partial wave in expression (3.12). The values of first few spherical Bessel functions are.

$$j_0(kr)=\frac{Sinkr}{kr},\ j_1(kr)=\frac{Sinkr}{(kr)^2}-\frac{Coskr}{kr},$$
$$j_3(kr)=\left(\frac{3}{(kr)^3}-\frac{1}{kr}\right)Sinkr-\frac{3Coskr}{(kr)^2}$$

The spherical harmonic function.

$$Y_{l,0}(\theta\ )\ =\ \frac{(2l+1)^{1/2}}{(4\pi)^{1/2}}(Cos\theta) \longrightarrow 3.15$$

where $P_l(Cos\theta)$ is the Legendre polynomial of order l. The square of the spherical harmonic function gives the angular dependence of the probability density. The values of the first few Legendre polynomials are

$$P_0(Cos\theta)=1, \qquad P_1(Cos\theta)=Cos\theta \qquad P_2(Cos\theta)=\frac{3Cos^2\theta-1}{2}$$

For incident neutron kinetic energy les than 10 Mev the only partial wave involved in scattering is the $l = 0$ or S – wave.

In the absence of a scattering potential eqn(3.12) can be written

$$\psi = R_0(r)Y_{0,0}(\theta) + \sum_{l=1}^{\alpha} R_l(r)Y_{l,0}(\theta) = \frac{Sinkr}{kr} + \left(e^{ikz} - \frac{Sinkr}{kr}\right) \longrightarrow 3.16$$

The average value of the quantity with in the brackets over all directions in space is zero. The first term corresponds to the spherically symmetric partial wave (s – wave).For S-wave scattering, only the first term is affected and can therefore be written as in the presence of scattering potential V(r) . We can write it as $\psi_S = \frac{U(r)}{r}$

Thus as $r \longrightarrow \alpha$, the solution u(r) assumes the form $cSin(kr + \delta_0)$, where c is an arbitrary constant and δ_0 is some phase angle. Thus the complete wave function outside the scattering potential is

$$\psi = \frac{cSin(kr + \delta_0)}{r} + \left(e^{ikz} - \frac{Sinkr}{kr}\right)$$

$$= e^{ikz} + \frac{c}{r}\left(\frac{e^{ikr}e^{i\delta_0} - e^{-ikr}e^{-i\delta_0}}{2i}\right) - \frac{1}{kr}\left(\frac{e^{ikr} - e^{-ikr}}{2i}\right)$$

$$\psi = e^{ikz} + \frac{e^{ikr}}{r}\left(\frac{ce^{i\delta_0} - 1/k}{2i}\right) - \frac{e^{-ikr}}{r}\left(\frac{ce^{-i\delta_0} - 1/k}{2i}\right) \longrightarrow 3.17$$

This scattering wave must contain no incoming wave therefore we can write the coefficient of e^{-ikr} as zero.

$$ce^{-i\delta_0} - 1/k = 0$$

$$\Rightarrow c = \frac{1}{k}e^{i\delta_0}$$

substitute above values in eq(3.17) then

$$\psi = e^{ikz} + \frac{e^{ikr}}{r}\frac{e^{2i\delta_0} - 1}{2ik} \longrightarrow 3.18$$

comparing this with the standard solution (3.9) we have

$$f(\theta) = \frac{e^{2i\delta_0} - 1}{2ik} = \frac{e^{i\delta_0}}{k}\frac{e^{i\delta_0} - e^{-i\delta_0}}{2i} = \frac{e^{i\delta_0}}{k} Sin\,\delta_0 \longrightarrow 3.19$$

The total elastic scattering cross section is

$$\sigma_0 = 2\pi\int_0^\pi \frac{Sin^2\delta_0}{k^2} Sin\,\theta d\theta = \frac{4\pi}{k^2} Sin^2\delta_0 = 4\pi\lambda^2 Sin^2\delta_0 ....3.20$$

This analysis here is carried through only for $l = 0$ scattering

Higher orbital angular momentum waves also have to be considered at higher energies. The total cross-section can be written as a sum of partial cross-section , one for each l-wave. The partial cross-section

$$\sigma_l = 4\pi\lambda\!\!\!^{-2}(2l+1)Sin^2\delta_l \longrightarrow 3.21$$

Scattering length:

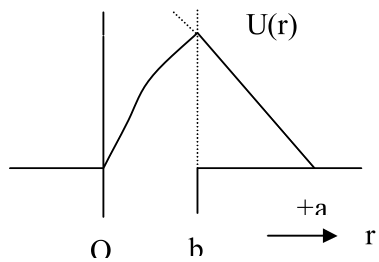
For neutrons of very low energy scattered by free protons, $\lambda\!\!\!^{-}$ is very large and hence K is very small. It can be seen from eqn(3.19) that as K$\longrightarrow$ 0 must also approach zero. Otherwise $f(\theta)$ would become infinite. Thus for low energy neutrons $f(\theta)$ can be written as

$$f_0 = \lim_{\delta_0 \to 0} \frac{e^{i\delta_0} Sin\,\delta_0}{k} = \frac{\delta_0}{k} = -a \longrightarrow 3.22$$

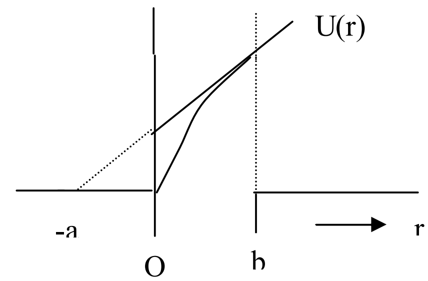
where the quantity + a is called the scattering length in the convention of Fermi and Marshall. Hence for low energy neutrons.

$$U(r) = c(kr + \delta_0) = ck(r - a) \longrightarrow 3.23$$

this is the equation of a straight line intersecting the r-axis at r = a, and is obtained by extrapolating the radial wave function u(r) from the point just beyond the range of the nuclear forces.



3.3.a Positive scattering length



3.3.b.Negative scattering length (unbound state)

Scattering from a potential giving a bound state produces a positive ‘a’. If the potential gives only a virtual state, the slope of the inner wave function at r = b is positive and ‘a’ is negative.

From eqn(3.20) & (3.22) the zero energy scattering cross section becomes

$$\sigma_0 = 4\pi a^2 \longrightarrow 3.24$$

This is identical with the scattering cross-section of an impenetrable sphere of radius ‘a’, in the limit of zero energy. The measurement of σ_0 determines the magnitude of the scattering length ‘a’ but not its sign.

Determination of the phase shift δ_0:

We shall now attempt to determine the phase shift δ_0 for low energy neutron - proton scattering by solving also the schrodinger’s equation in the region where the interaction between the two particles takes place. For this we make the simple assumption of a square well for the nuclear potential. Inside the well of depth V_0 and radius ‘b’ the radial wave equation for particles whose total energy has the positive value E is

$$\frac{d^2u}{dr^2}+\frac{M}{\hbar^2}(E+V_0)u(r)=0 \longrightarrow 3.25$$

Inside the square well this equation has the simple solution

$$u(r)=ASink_1r, where \quad k_1=\frac{\sqrt{M(E+V_0)}}{\hbar} \longrightarrow 3.26$$

Outside the square well, the solution can be written as

$$u(r)=B\,Sin(kr+\delta_0) \longrightarrow 3.27$$

At the edge of the rectangular well(r = b), the two solutions and their derivatives with respect to r must be continuous.

$$\therefore A\,Sink_1b=B\,Sin(kb+\delta_0)$$
$$k_1ACosk_1b=BkCos(kb+\delta_0)$$
$$Hence \quad k_1Cotk_1b=KCot(kb+\delta_0) \longrightarrow 3.28$$

From Deuteron problems

$$K \quad Cotkb=-\alpha$$

Which describes the binding energy B of the deuteron in terms of the same rectangular well (v_o ,b)

Here $$K=\frac{\sqrt{M(V_0-B)}}{\hbar} \qquad and \qquad \alpha=\frac{\sqrt{MB}}{\hbar}$$

For low energy neutrons (E << V_o), we may assume K = k_1 (as V_o >> B), hence the wave function u(r) inside the well is nearly the same for the deuteron and the n-p scattering system. Thus for approximation can be written as

$$\frac{\sqrt{ME}}{\hbar}Cot\left(\frac{\sqrt{ME}}{\hbar}b+\delta_0\right)=-\frac{\sqrt{MB}}{\hbar}$$

As the scattering length 'a' is much larger than the range 'b' of the potential , thus for very low energy neutrons kb can be neglected in comparison to δ_0

$$\therefore \quad Cot\delta_0 = -\sqrt{\frac{B}{E}}$$

$$or \qquad Sin\delta_0 = \frac{\sqrt{E}}{E+|B|} \longrightarrow 3.29$$

substituting this value of $\sin^2\delta_0$ in equation (3.20) We obtain the approximate value of total scattering cross-section as

$$\sigma = 4\pi\hbar^2 \frac{E}{(E+|B|)^2}$$

$$= 4\pi\hbar^2 \left(\frac{E}{E+|B|}\right)\frac{1}{E+|B|}$$

$$= \frac{4\pi\hbar^2}{M}\frac{1}{E+|B|} \longrightarrow 3.30$$

3.4.MESON THEORY OF NUCLEAR FORCES(Yukawa Potential):-

Any attractive force between two particles is regarded as the exchange of an attractive property. In molecular bonds, the valence electrons are exchanged. Now the natural question is "what is exchanged in nuclear bonds?" Is it electron , No it can not be ; because it leads to a very weak interaction. Yukawa, in 1935, proposed that nuclear forces are due to an exchange of particles of intermediate mass, known as mesons (Yukawa particle). Yukawa took the analogy from the quantum field theories of electromagnetic field in which photon exchange takes place and gravitational field in which exchange of graviton is assumed. Both these field particles have zero rest mass, but the nuclear field particles has finite rest mass. This is because of the difference that nuclear force is short range force while others are not. The rest mass m_π of the field particle may be computed as follows.

When one nucleon exerts force on the other, meson is created and the creation of meson violates the conservation of energy by amount ΔE corresponding to meson rest mass i.e.

$$\Delta E = m_{\pi} c^2 \longrightarrow 3.31$$

The duration of excursion of meson, Δt is given by uncertainty principle,

$$\Delta t \approx \frac{\hbar}{\Delta E} \longrightarrow 3.32$$

In this time meson can cover a distance R_0, given by

$$R_0 = c\Delta t = \frac{c\hbar}{\Delta E} \ldots\ldots\ldots\ldots\ldots 3.33$$

$$R_0 = \frac{c\hbar}{m_{\pi} c^2} = \frac{\hbar}{m_{\pi} c}$$

R_0 is the range of nuclear force and if we put $R_0 = 1.4$fermi

$$m_{\pi} = \frac{\hbar}{R_0 c} = \frac{0.045 \times 10^{-34}}{1.4 \times 10^{-15} \times 3 \times 10^{8}.} \ldots\ldots\ldots\ldots\ldots\ldots 3.34$$

$$m_{\pi} = 270 m_e$$

where m_e *is the rest* mass of the electron

$$= 9.109 \times 10^{-31}\, Kg$$

A search for Yukawa particle started soon after its hypothesis and in 1947, Powell discovered Π – meson in cosmic radiation and has a mass 273 m_e This particle, Π – meson (pion), is exact Yukawa particle. The pions are of three kinds positive (Π^+) ,negative(Π^-) and neutral(Π^o) all with intrinsic spin S = 0 . The force field between two proton or two neutrons is carried by a neutral pion while between a neutron and proton by a charged pion. In the latter case conversion of one nucleon to the other takes place. The situation is illustrated as

$$P \longleftrightarrow P + \pi$$
$$n \longleftrightarrow n + \pi$$
$$P \longleftrightarrow n + \pi^+$$
$$n \longleftrightarrow P + \pi^- \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.35$$

To interpret the nucleon – nucleon scattering data interms of a potential function, let us compare the meson theory with the quantum theory of electro-magnetic interactions. To obtain a pion wave equation we express the total energy E of a pion in terms of the pion rest mass energy $m_{\Pi} c^2$ and momentum 'P' as

$$E^2 = P^2 C^2 + m_{\Pi}^2 c^4 \longrightarrow 3.36$$

The energy E and momentum component P are represented by the operator form

$$E = i\hbar\, \partial/\partial t, \qquad P_x = -i\hbar\, \partial/\partial x, \qquad P_y = -i\hbar\, \partial/\partial y, \qquad P_z = -i\hbar\, \partial/\partial z$$

Now eqn(3.36) can be written as

$$-\hbar^2 \frac{\partial^2}{\partial t^2} = -\hbar^2 \nabla^2 c^2 + m_\pi{}^2 c^4 \longrightarrow 3.37$$

It Φ is pion wave function, then wave equation for pion takes the form

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{m_\pi{}^2 c^2}{\hbar^2}\right)\phi = 0$$

This is Klein Gordon equation for a free particle of spin 0 if we set $m_{\Pi} = 0$, we get

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\phi = 0 \longrightarrow 3.38$$

This is the wave equation for electromagnetic field. Now the simplest form of electromagnetic field is electrostatic field. The corresponding equation is obtained by putting

$$\frac{\partial \phi}{\partial t} = 0, i.e., \nabla^2 \phi = 0, Laplace\ \ equation$$

The identical equation for the meson field is

$$\left(\nabla^2 - \frac{m_\pi{}^2 c^2}{\hbar^2}\right)\phi = 0$$

This is in absence of any source of mesons, but in the presence of source, the equation should resemble with poisson's equation i.e. it should have the form

$$\left(\nabla^2 - \frac{m_\pi^{\ 2} c^2}{\hbar^2}\right)\phi = 4\pi g \delta(r)$$

$$\left(\nabla^2 - \mu^2\right)\phi = 4\pi g \delta(r) \quad \text{..........................3.39}$$

Where $\mu = \frac{m_\pi c}{\hbar} = 1/R_0$. The factor 'g' plays the same role as the charge 'e' plays in the case of electrostatic field, and is a measured of nuclear field and known as mesonic charge. $\delta(r)$ is Dirac delta function $\{\delta(r) = 1 \ \ at \ r = 0 \ \ , \delta(r) = 0 \ \ at \ finite \ \ r\}$. The solution of equation (3.39) comes out to be

$$\phi = -g \frac{e^{-\mu r}}{r} \longrightarrow 3.40$$

Therefore, the meson potential will be

$$V(r) = g\phi = -g^2 \frac{e^{-\mu r}}{r}$$

The shape of the potential is shown in fig.3.4

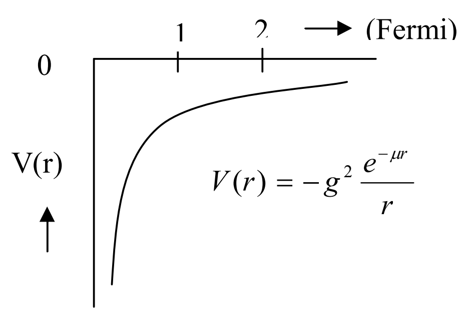


Fig . 3.4. Shape of the Potential

3.5.Summary: The P-P scattering is the only way to get direct evidence on proton-proton force. There are two essential differences between p-p and n-n scattering. First, the p-p scattering is caused not only by nuclear force but also by the coulomb force. Second, the scattering and scattered particles are identical and obey the pauli exclusion principle and therefore, the wave function describing the two protons must change sign on the interchange of the two particles. Yukawa, in 1935, proposed that nuclear forces are due to an exchange of particles of intermediate mass, known as mesons (Yukawa particle). Yukawa took the analogy from the quantum field theories of electromagnetic field in which photon exchange takes place and gravitational field in which exchange of graviton is assumed. Both these field particles have zero rest mass, but the nuclear field particles has finite rest mass.

Keywords:
Scattering, Phase shift, radial wave

Self assessment questions:

1. Explain proton-proton scattering at low energies
2. Discuss neutron-proton scattering at low energies
3. Explain the meson theory of nuclear forces.

Text books

1. Nuclear physics by D.C.Tayal , Himalaya publishing company,Bombay.
2. Nuclear physics by R.C.Sharma, K.Nath&co, Merut
3. Nuclear physics by S.B.Patel

Unit 1

Lesson 4

NUCLEAR REACTIONS

The objectives of the lesson are to explain the following:

4.1 Introduction

4.2 Types of Reactions

4.3 Conservation laws

4.4 Nuclear Kinematics

4.5 Nuclear Energy

4.6.Summary

4.1 Introduction : A considerable part of our present knowledge of the nuclear structure comes from experiments in which a chosen nucleus is bombarded with different projectiles, such as protons, neutrons, deuterons or α - particles. When these particles come close enough to interact with the target nuclei, either elastic or inelastic scattering may take place or one or more particles which are altogether different may be knocked out of the nucleus, or the incident particle may perhaps be captured and a gamma ray emitted. When the mass number and/ or atomic number of the target nuclei changes after the bombardment, we say that a nuclear reaction has taken place.

Nuclear Reaction: Nuclear reaction is the process of strong interaction of an atomic nucleus with an elementary particle resulting in the formation of a new nucleus and one or more new particles.

A nuclear reaction can be represented as follows

$$X + x \longrightarrow Y + y$$

Where X is the target nucleus, 'x' the incident or projectile particle, Y the product nucleus and 'y' the out going or detected particle.

Nuclear reaction was discovered by Rutherford, this reaction an be represented as follows.

$$_7N^{14} + {_2He^4} \longrightarrow {_9O^{17}} + {_1H^1}$$

In case of a nuclear reaction

1. The total atomic number of incident particle and target nucleus before the reaction must be equal to the total atomic number of the products after reaction.
2. The total sum of atomic mass numbers on both sides of the equation must be equal

Importance of Nuclear Reaction Equations:

The nuclear equations have many practical applications. Few of them are

1. We calculate the different elements atomic masses accurately
2. They help to discover and identify the new isotopes
3. They provide the experimental verification of Einstein's mass-energy relation ($E = mc^2$)
4. They provide the information to predict other possible reactions.

4.2 Types of Nuclear Reactions:

1. Elastic scattering
2. Inelastic scattering
3. Disintegration
4. Photo disentegration
5. Radiative capture
6. Direct Reaction
7. Heavy ion reactions

8. Spontaneous decay
9. Spallation Reactions
10. High energy Reactions

(1)Elastic scattering: In this the incident particle strikes the target and leaves without energy loss. But here direction of motion is altered. Scattering of alpha particles in gold is a good example of this process

$$\text{Ex. } {}_2He^4 + {}_{79}Au^{197} \longrightarrow {}_{79}Au^{197} + {}_2He^4$$

(2)Inelastic scattering: whenever scattered particle has excess energy to form elastic collision with nucleus then scattered particle may loss K.E.. This energy is being corresponding increase in the internal energy of the nucleus as which is excited to a higher quantum state. This excess energy is later radiated away in the form of a quantum

$$\text{Ex. } {}_3Li^7 + {}_1H^1 \longrightarrow ({}_3Li^7)^* + {}_1H^1$$

* is used to indicate that after scattering the nucleus is left in an excited state.

(3) Disintegration: On striking the target nucleus the incident particle is absorbed and a different particle is ejected. The product nucleus differs from target nucleus. The incident particle may be alpha particle, proton, neutron etc.The product particle may be a charged particle or a neutron

$$\text{Ex. } {}_7N^{14} + {}_2He^4 \longrightarrow {}_8O^{17} + {}_1H^1$$

(4)Photodisintegration: In this γ –rays are absorbed by the target nucleus, exciting it to a higher quantum state. If the energy is high enough, one or more particles may be liberated.An example is

$${}_1H^2 + \gamma \longrightarrow {}_1H^1 + {}_0n^1$$

(5) Radiative Capture: A particle may combine with a nucleus to produce a new nucleus or a compound nucleus which is in an excited state. The excess energy is emitted in the form of γ – ray photons. Ex. ${}_{12}Mg^{26} + {}_1H^1 \longrightarrow ({}_{13}Al^{27})^* \longrightarrow {}_{13}Al^{27} + \gamma$

(6)Direct Reactions: In this we have two categories

a)Pick up reactions

b)Stripping reactions

a)Pick up reaction: A collision of an incident particle with the nucleus may immediately pull one of the nucleons out of the target nucleus.

Ex. ${}_{8}O^{16} + {}_{1}H^{1} \longrightarrow {}_{8}O^{15} + {}_{1}H^{3}$

b)Stripping reaction: In this a bombarding particle which is composed of more than one nucleon may loss one of them to the target.

Ex. ${}_{29}Cu^{63} + {}_{1}H^{2} \longrightarrow {}_{29}Cu^{64} + {}_{1}H^{1}$

(7) Heavy ion Reactions: Nuclear reactions induced by heavy ions exhibit the characteristics both of compound nucleus and of the stripping and pickup mechanisms

Ex. ${}_{7}N^{14} + {}_{82}Pb^{207} \longrightarrow {}_{7}N^{13} + {}_{82}Pb^{208}$

(8)Spontaneous decay: Beta decay , alpha decay processes may be regarded as this type of nuclear reactions. In these reactions, the total energy of the system is not under the experimental's control. Here incident particle is not necessary.

${}_{84}Po^{212} \longrightarrow {}_{82}Pb^{208} + {}_{2}He^{4}$ (α decay)

this reaction takes place due to instability.

${}_{1}H^{3} \longrightarrow {}_{2}He^{3} + {}_{-1}e^{0}$ (β – decay)

(9)Spallation Reaction: On capture of a incident particle, a heavy nucleus has sufficient energy for the ejection of several particles Such a reaction is known as spallation reaction eg. The nuclear fission.

Ex. ${}_{92}U^{235} + {}_{0}n^{1} \longrightarrow {}_{40}Zr^{98} + {}_{52}Te^{136} + 2\,{}_{0}n^{1}$

(10)High energy reaction: Energy range is about 150Mev then spallation process merges in to new kind of reaction in which new particles (mesons, strange particles) are produced along with neutrons and protons.

4.3 CONSERVATON LAWS: Certain quantities must be conserved in any nuclear reaction.

Various conservation laws which valid in ordinary nuclear interactions are given below.

1. Conservation of Energy
2. Conservation of momentum
3. conservation of Angular momentum
4. Conservation of Charge
5. Conservation of Nucleons
6. Conservation of Spin
7. Conservation of Parity
8. Conservation of Isotopic spin

1. **Conservation of Energy:** The total energy of the products, including both mass energy and kinetic energy of the particles plus the energy involved must be equal to the mass energy of the initial ingredients plus the kinetic energy of the bombarding particle.

2. **Conservation of Momentum:** The total linear momentum of the products must be equal to the linear momentum of the bombarding particle.(the target nucleus in ordinarily takes to be rest)

3. **Conservation of Angular momentum:** The total angular momentum j comprising the vector sum of the intrinsic spin angular momentum 's' and relative orbital angular momentum 'l' of the products must be equal to the total angular momentum of the initial particles.

4. **Conservation of Charge:** The total electric charge of the products must be equal to the total electric charge of initial particles.

5. **Conservation of Nucleons:** The law of conservation of nucleons states that the nucleons can neither be created nor be destroyed so that the number of nucleons minus the number of anti nucleons in the universe remains constant.

6. **Conservation of Spin:** The spin character of a closed system can not change i.e. the statistics remains same as that existed before reaction.

7. **Conservation of Parity:** The parity of the system determined by the target nucleons and bombarding particle must be conserved throughout the reaction.The total parity of the system is the product of intrinsic particles of the target nucleus and bombarding nucleus. No violation of parity has been observed in a nuclear reaction(Strong nuclear forces). Although parity does not appear to be conserved in weak interactions.

8. **Conservation of Isotopic spin:** The invariance of the nuclear Hamiltonian function towards the charge character of the nucleons can be expressed analytically as an invariance towards rotational shifts of the axes in isotopic spin space, and there should correspondingly exist a conservation law for the isotopic spin of a nuclear system.

4.4 NUCLEAR KINEMATICS:

At any energy, the conservation of energy and momentum imposes certain restrictions on the reactions. These restrictions are called kinematic reactions and this mathematical method is known as kinematics. Consider a nuclear reaction.

$$X + x \longrightarrow Y + y$$

Where X - target nucleus

x - bombarding particle

Y – Product nucleus

y - product particle

In this reaction we assumed that target nucleus is at rest so it has no kinetic energy.

Since the total energy is conserved in nuclear reaction, we get

$$M_Xc^2 + (E_x + m_x c^2) = (E_Y + M_Y c^2) + (E_y + m_y c^2) \ldots\ldots\ldots\ldots\ldots\ldots \quad (4.1)$$

Where m_x = mass of incident particle

M_X = Mass of target nucleus

m_y = mass of product particle

M_Y = mass of product nucleus

From (4.1),

$$(M_X + m_x - M_Y - m_y)c^2 = E_Y + E_y - E_x = Q \ldots\ldots\ldots\ldots\ldots\ldots \quad (4.2)$$

Here we introduced a quantity Q which represents the difference between the kinetic energy of the products of reaction and that of the incident particle.

The quantity Q is called the energy balance of the reaction or more commonly Q- value of the reaction depending upon Q we have two types of reactions.

1. Exoergic reactions
2. Endoergic reactions

1.Exoergic reactions: If Q is +ve then that type of reaction is known as exoergic reaction. This takes place only if sum of the masses of incident particle and target nucleus is greater than that of masses of the product nuclei. K.E of the product nuclei being greater than that of the incident particle.

2.Endoergic reaction: If Q is -ve the reaction is said to be endoergic reaction. i.e. energy must be supplied usually as a K.E. of the incident particle.

Q-Equation:

$$Q = E_Y + E_y - E_x \longrightarrow 4.3$$

To measure a Q – value in an experiment we first measure the bombarding energy E_x and the energy of the ejected particle E_y at some specified angle θ. In eqn (4.3) E_Y – recoil energy of the product nucleus. It's value is small and hard to measure.

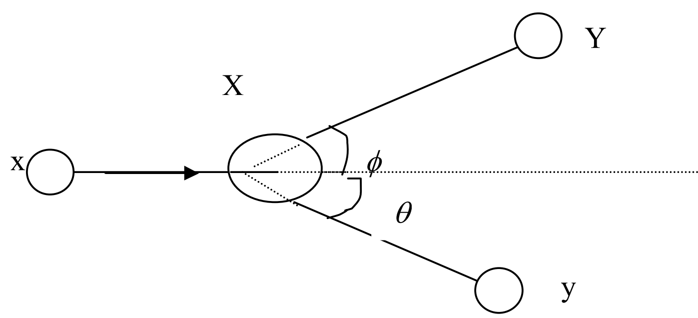


Fig: 4.1 Schematic diagram of a nuclear reaction

Thus applying the law of conservation of momentum we have.

$$m_x v_x = M_Y V_Y \, Cos\Phi + m_y v_y \cos\theta \longrightarrow (4.4)$$

and $$M_Y V_Y \, Sin\Phi = m_y v_y Sin\,\theta$$

Here v_x = velocity of incident particle

V_y = velocity of ejected particle

V_Y = velocity of product nucleus

From eqn(4.4), $M_Y V_Y \, Cos\Phi = m_x v_x - m_y v_y \cos\theta$

$$M_Y V_Y \, Sin\Phi = m_y v_y Sin\,\theta \qquad \ldots\ldots\ldots\ldots 4.4(a)$$

From eqn 4.4(a), we get

$$M_Y{}^2 V_Y{}^2 = m_x{}^2 v_x{}^2 + m_y{}^2 v_y{}^2 - 2 m_x v_x m_y v_y Cos\theta \ldots\ldots\ldots 4.4(b)$$

Since $E_x = \frac{1}{2} m_x v_x{}^2$, $E_y = \frac{1}{2} m_y v_y{}^2$, $E_Y = \frac{1}{2} M_Y v_Y{}^2$

Substituting the values of V_Y^2, v_x^2, v_y^2 from the above eqns, into eqn 4.4(b) we get the resultant equation as shown below

$$2E_Y M_Y = 2E_x m_x + 2E_y m_y - 4(m_x m_y E_x E_y)^{1/2} Cos\theta$$

$$or \quad E_Y = E_x \frac{m_x}{M_Y} + E_y \frac{m_y}{M_Y} - \frac{2}{M_Y}(m_x m_y E_x E_y)^{1/2} Cos\theta$$

$$Q = E_Y + E_y - E_x$$

$$= E_x \frac{m_x}{M_Y} + E_y \frac{m_y}{M_Y} - \frac{2}{M_Y}(m_x m_y E_x E_y)^{1/2} Cos\theta + E_y - E_x$$

$$Q = E_x \frac{(m_x - M_Y)}{M_Y} + E_y \frac{(m_y + M_Y)}{M_Y} - \frac{2}{M_Y}(m_x m_y E_x E_y)^{1/2} Cos\theta \text{............................} (4.5)$$

Equation (4.5), represents the Q-value reaction. It gives the desired relation between the energy released and the measured quantities E_x, E_y and θ for a special case, when we are observing the out going particle if at 90^0 to a collimated beam of projectile the above eqn (4.5) reduces to

$$Q = E_Y \frac{(m_y + M_Y)}{M_Y} + E_x \frac{(m_x - M_Y)}{M_Y}$$

$$= E_Y\left(1 + \frac{m_y}{M_Y}\right) - E_x\left(1 - \frac{m_x}{M_Y}\right) \text{......................} 4.5(a)$$

By using modern techniques, we measure Q values for nuclear reaction producing charged particles, to an accuracy of 1 part in a thousand or better.

We now find the general solution of the Q – equation

From eqn(4.5)

$$Q = E_x \frac{(m_x - M_Y)}{M_Y} + E_y \frac{(m_y + M_Y)}{M_Y} - \frac{2}{M_Y}(m_x m_y E_x E_y)^{1/2} Cos\theta$$

$$QM_Y = E_x(m_x - M_Y) + E_y(m_y + M_Y) - 2(m_x m_y E_x E_y)^{1/2} Cos\theta$$

$$QM_Y + E_x(M_Y - m_x) = E_y(m_y + M_Y) - 2(m_x m_y E_x E_y)^{1/2} Cos\theta$$

$$\frac{QM_Y + E_x(M_Y - m_x)}{(m_y + M_Y)} = E_y - \frac{2(m_x m_y E_x E_y)^{1/2}}{(m_y + M_Y)} Cos\theta \text{..........................(4.6)}$$

$$\frac{QM_Y - E_x(m_x - M_Y)}{(m_y + M_Y)(E_Y)^{1/2}} = (E_Y)^{1/2} - \frac{2(m_x m_y E_x)^{1/2}}{(m_y + M_Y)} Cos\theta \text{.......................(4.7)}$$

Now

$$\left[(E_y)^{1/2} - \frac{(m_x m_y E_x)^{1/2}}{(m_y + M_Y)} Cos\theta\right]^2 = E_Y - \frac{2(m_x m_y E_x E_y)^{1/2}}{(m_y + M_Y)} Cos\theta + \frac{(m_x m_y E_x)}{(m_y + M_Y)^2} Cos^2\theta$$

$$E_y - \frac{2(m_x m_y E_x E_y)^{1/2}}{(m_y + M_Y)} Cos\theta = \left[(E_Y)^{1/2} - \frac{2(m_x m_y E_x)^{1/2}}{(m_y + M_Y)} Cos\theta\right]^2 - \frac{(m_x m_y E_x)}{(m_y + M_Y)^2} Cos^2\theta$$

From (4.7)

$$.\frac{QM_Y + E_x(M_Y - m_x)}{(m_y + M_Y)} = \left[(E_y)^{1/2} - \frac{2(m_x m_y E_x)^{1/2}}{(m_y + M_Y)} Cos\theta\right]^2 - \frac{(m_x m_y E_x)}{(m_y + M_Y)^2} Cos^2\theta$$

$$\left[(E_y)^{1/2} - \frac{2(m_x m_y E_x)^{1/2}}{(m_y + M_Y)} Cos\theta\right]^2 = \frac{QM_Y + E_x(M_Y - m_x)}{(m_y + M_Y)} + \frac{(m_x m_y E_x)}{(m_y + M_Y)^2} Cos^2\theta$$

$$\left(\sqrt{E_y} - u\right)^2 = v + u^2 \Rightarrow \left(\sqrt{E_y} - u\right) = \pm\sqrt{v + u^2}$$

$$\sqrt{E_y} = u \pm \sqrt{u^2 + v} \text{..........................(4.8)}$$

$$where \quad u = \frac{\sqrt{m_x m_y E_x} Cos\theta}{m_y + M_Y}, \qquad v = \frac{QM_Y + E_x(M_Y - m_x)}{m_y + M_Y} \text{...........(4.9)}$$

<u>Exoergic reaction:</u> $(Q > 0)$ these reactions are possible even for $E_x = 0$. Thus for $E_x \rightarrow 0$.

From eqns (4.8) and (4.9) $E_y = \dfrac{QM_Y}{m_y + M_Y} (Q > 0)$ …………..4.10

The kinetic energy E_Y is the same for all angles θ. From this we observe the total momentum is effectively zero when $E_x \to 0$

In this case $Q = E_Y + E_y$ and $\theta + \Phi = 180^0$

Eqn (4.7) has two solutions. One is positive and another is negative. Hence 'E_y' to be single valued $Q > 0$ and $M_Y > m_x$ generally projectile 'x' is lighter than the product nucleus Y. Thus υ is +ve for all values of the bombarding energy and 'E_y' depends on cosθ and it has smallest value in the backward direction. $\theta = 180^0$

2)Endoergic Reactions: Q value is -ve then this reaction takes place. Generally these reactions are not possible because when $E_x \to 0$, eqn(4.7) gives $u^2 + v$ = -ve. Then $\sqrt{E_Y}$ is imaginary.

Threshold energy: The threshold energy is defined as the minimum kinetic energy of the incident particle which will initiate an endoergic reaction. This is expressed by E_{th}

This reaction is possible when E_x is large enough to make $u^2 + v = 0$

Then

$$E_x = -Q\left[\frac{m_y + M_Y}{m_y + M_Y - m_x - \left(\frac{m_x m_y}{M_Y}\right)Sin^2\theta}\right] \text{...........4.11}$$

E_x is minimum at θ = 0 and this is called threshold energy.

Therefore, By using relations

$$(E_x)_{th} = -\frac{Q(m_y + M_Y)}{m_y + M_Y - m_x}$$

$$Q = (M_X + m_x - M_Y - m_y)c^2$$

$$\textit{In this case} \quad M_X \succ\succ \frac{Q}{c^2} \qquad \textit{then}$$

$$(E_x)_{th} = -Q\frac{(m_x + M_X)}{M_X}$$

Thus we see that of threshold of the reaction particles first appear in $\theta = 0$ direction with the K.E.

$$E_y = u^2 = (E_x)_{th}\left(\frac{m_x m_y}{(m_y + M_Y)^2}\right)$$

As the bombarding energy is raised, particle 'y' begins to appear at $\theta \succ 0$

4.5. Nuclear Energy

A Nuclear reactor is a device or apparatus in which nuclear fission is produced under a self sustaining controlled nuclear chain reaction.

Essential components of a nuclear reactor are given below.

1. Fuel
2. Reactor core
3. Reactor reflector
4. Reactor moderator
5. Reactor coolant
6. Reactor control materials
7. Reactor Sheilding

1) Fuel: The material containing the fissible isotope is called the reactor fuel. Generally U^{235} is used as fuel in many reactions.

2) Reactor core: This is the main part of the reactor. Reactor cores generally have a shape approximately a right circular cylinder with a diameter of few metres. In general reactor core have fuel elements, moderator control rods and cooling material housed in a pressure vessel.

3) Reactor reflector: The region surrounding the reactor core is known as reflector. It reflect back some of the neutrons which are leak out from the surface of the core.The material of the reflector is the same as that of moderator.

4) Reactor moderator: Reactor moderator is used to slow down the fast neutrons. Commonly used moderators are a)Ordinary water b) heavy water c) graphite (carbon) and d) Beryllium oxide. Heavy water is the best moderator.

5)Reactor Coolant: The material used to remove heat produced by fission as fast as it is liberated is known as reactor coolant.

Ex. Ordinary and heavy water, organic liquids(hydrocarbons), liquid metals and gases.

6) Reactor control materials: In a reactor it is very essential to control the fission process otherwise the chain may become explosive and consequently the reactor will be damaged.

7) Reactor Sheilding: In a nuclear reactor various types of rays are emitted. These rays may harm the persons working near the reactor. Hence thick walls of cement and concrete are constructed around the reactor which are known as shields.

Working: Some neutrons are remain in the reactor. To operate the reactor, all the controlling rods except one are taken out . Other controlling rods taken out slowly until the intensity of neutrons begins to increase. The reactor now works at a constant level by adjusting the single control rod.

Types of Reactors: Reactors are classified in to three types

1. Research Reactors
2. Production Reactors
3. Power Reactors

1) Research Reactors: Research reactors are used primarily to supply neutrons for physical research and radio isotope manufacture. In these reactors the total energy liberated is comparatively small, since they operate at a low level of reactivity.

We shall describe here five main types of research reactors.

1. Graphite – Moderated Research Reactors
2. Water – Boiler type Reactors
3. Swimming pool Reactors
4. Light – Water – Moderator, tank type Reactor
5. Heavy – Water – Moderator, tank type Reactor

1) Graphite – Moderated Research Reactors: This is the first reactor, which we built and operated it. This first reactor now called CP –1 (Chicago pile –1) was put into operation in 1942. This was operated without coolant flow at a power level of 200 watts. Similar assemblies with air as coolant, like the GLEEP (Graphite low energy experimental pile) and BEPO (British experimental pile), BNL (Brookhaven National Laboratory) are well known examples.

2) Water – Boiler type Reactors: The water boiler is usually a homogeneous mixture of a highly enriched uranium salt dissolved in ordinary water. The first reactor of this type was

LOPO (for low power operation)

HYPO (for high power operation)

SUPO (super power operation)

WBNS (Water Boiler neutron source)

ARR (Aromoue Research Reactor)

KEWB (kinetic Exponential water Boiler)

3) Swimming pool Reactors: It is light water moderated heterogeneous reactor which consists of a lattice of enriched uranium fuel immersed in a large pool of water. The water in the pool plays the role of moderator, coolant and shield BSR is the first swimming pool reactor which is completed in 1950. The Indian Reactor Apsara belongs to this type. It has maximum power level of 1 Mw. The other reactors are NRLR (Naval), Battel, (RR) swiss reactor etc.

4) Light – Water – Moderator, tank type Reactor: This reactor is similar to pool reactor in principle except that the core is suspended in a deep cylindrical tank of water and can not be moved. First reactor was built in 1952. and it was name d as MTR (Material Testing Reactor). In this reactor neutrons flux was increased . The other reactors are ETR, WTR, HFR (High flux reactor)

5) Heavy – Water – Moderator, tank type Reactor: The desirability of obtaining heavy water for use as a neutron moderator lies in the extremely small thermal absorption cross section deuterium. Because of this property the neutron economy in a heavy water system is greatly improved. The first reactor is CP-3 (Chicago pile –3) CIRUS or CIR is India's second research reactor. CIR - Canadian Indian Reactor. Power is 40 MW Zerlina – India's third Research reactor. It's name stands for Zero energy lattice investigation nuclear assembly - In this natural uranium is used as fuel.

Purnima - It is India's first experimental zero energy fast reactor. In this fuel is plutonium oxide.

R – 5 - Thermal research reactor with a nominal power of 100MW

II)Production Reactors: The purpose of a production reactor is sto convert fertile into fissible material. In these reactors uranium is used as fuel. Graphite as a moderator and water as coolant. The conversion efficiency of a production reactor is high when we use heavy water instead of graphite as a moderator.

III)Power Reactors: The primary purpose of a power reactor is the utilization of the fission energy produced in the reactor core and to convert it into useful power. During the chain reaction large amount of heat being generated in the reactors. Hence these reactors are used in power production and propulsion. Water moderator reactor could not be used if the tiny of the reactor core was high because water becomes highly corrosive fluid at high temperature. Alloy's of some metals were develop which could resist higher temperatures. In order to keep the water coolant in contact with reactor core at low temperatures. Water is keep at sufficient pressure to prevent boiling. This water is circulated through the core through a heat exchanger and is brought back to the core in a cycle of operation. Steam is generated in the heat exchange in a secondary circuit. This stem is used to a tubine eneg.

1)Pressurised water reactors: This is a heterogeneous reactor(it means that fuel is concentrated in plates) rods or hollow cylinders which are distributed in a regular pattern with in the moderator.. Homogeneous means the fuel and moderator are mixed in the form of a solution . In heterogeneous reactor fuel is enriched uranium light water as a moderator and coolant.

2) Boiling Water Reactor: The boiling water reactor is a small nuclear power plant designed to allow stem to be generated directly in the reactor core. This uses light water as moderator coolant . Boiling Nuclear super heat (BONUS) is one of the Boiling water reactors.

3)Heavy water moderated reactor: The advantage of heavy water is the total cost of the core and cost of the consumed fuel are both less in the heavy water moderated reactor. Because natural uranium is used as a fuel and the conversion ration for the regeneration of fissile material can be close to unity. Both factors decrease the cost of the power produced. The first reactor is Nuclear power demonstration reactor (NPDR) In India we have this type of reactor at kota (430MW) Kalpakam (470MW), Narora (440MW)

4)Organic Moderated reactor: This is the another method to achieve the high temperatures at moderate pressures in a thermal reactor. In this we use organic liquid containing a high hydrogen content and of is high boiling point .The first reactor is MORE.(Organic moderated reactor testing experiment)
PNPF (Piqua Nuclear power facitity)

5)Gas cooled Reactor: In this the mean time air was employed as the coolant in the graphite moderated production reactors at wide scale. Helium, Beryllium , CO_2 are used as moderators.

6)Sodium Graphite Reactors: The high boiling point of liquid sodium makes it an excellent coolant for a high temperature reactor. High temperatures s can be achieved without high pressures. In this reactor graphite is used as moderator, sodium as coolant, and uranium as the fuel.

7)Liquid fuel Reactor: The homogeneous reactor experiment is to test the feasibility of maintaining a fission chain at high temperatures and pressures . The advantage of an aqueous homogeneous reactor are the high power density, low fuel inventory, continuous removal of fission products and radiation damage products, high degree of nuclear stability.

8)Fast reactor: The development of power reactor in which the bulk of fissions are produced by high energy neutrons is of importance in the civilian power reactor development programmes. Such reactors not only produce useful power. But also regenerate more fissile material than is consumed.

Cross – Section of a Nuclear Reaction: The nuclear cross- section gives an idea of the probability that a bombarding particle will interact with the target material.

Let N_t = target nuclei per unit area.

N_0 = number of incident particles sticking per second per unit area of target material

N = number of nuclei undergoing interaction per second

Now we define nuclear cross- section σ by

$$\sigma = \frac{N}{N_0 N_t} m^2 / nucleus.$$

$$N_t = 1 \quad then \quad \sigma = \frac{N}{N_0}$$

Hence nuclear cross section may be defined as the ratio of number of nuclei undergoing interaction to the number of incident particles sticking per unit area when target nuclei per unit allows one.

It is a quantitative measure of the probability of a given nuclear reaction. fraction of incident particles are large than probability that the process will occur is greater.

The unit of nuclear reaction cross-section is barn.

$$1\text{barn} = 10^{-28}\ \text{m}^2$$

Determination of Cross-section: Consider a sheet of thickness t and area of cross section A containing n nuclei per unit volume. Let N_0 be the number of particles in the incident beam. Here we assume that each particle interacts only once. Further, due to scattering, absorption etc. Let dN particles be reduced from the beam in passing through a thickness dt. Let N be the number of particles which cross the sheet.

Now

$$-\frac{dN}{N}=\frac{Aggregate\,cross-section}{target\ area}$$

$$=\frac{nAdx\sigma}{A}=ndx\sigma$$

Integrating, we get

$$-\int_{N_0}^{N}\frac{dN}{N}=n\sigma\int_{0}^{t}dx$$

$$-\left(\log_e N\right)_{N_0}^{N}=n\sigma[x]_0^t$$

$$-\log_e\left(\frac{N}{N_0}\right)=n\sigma t$$

$$\log_e\left(\frac{N}{N_0}\right)=-n\sigma t$$

$$N=N_0e^{-n\sigma t}$$

From this relation, we can calculate σ. Thus $\sigma=\frac{1}{nt}\log_e\frac{N_0}{N}$.

4.6.Summary: Nuclear reaction is the process of strong interaction of an atomic nucleus with an elementary particle resulting in the formation of a new nucleus and one or more new particles.

A nuclear reaction can be represented as follows

$$X + x \longrightarrow Y + y$$

The nuclear equations have many practical applications. Few of them are

5. We calculate the different elements atomic masses accurately
6. They help to discover and identify the new isotopes
7. They provide the experimental verification of Einstein's mass-energy relation ($E = mc^2$)
8. They provide the information to predict other possible reactions.

If Q is +ve then that type of reaction is known as exoergic reaction.

If Q is -ve the reaction is said to be endoergic reaction.

A Nuclear reactor is a device or apparatus in which nuclear fission is produced under a self sustaining controlled nuclear chain reaction.

Reactors are classified in to three types

4. Research Reactors
5. Production Reactors
6. Power Reactors

Keywords:
Elastic scattering, Radiative capture, Spallation reactions, stripping reactions, exo-ergic, endo-ergic reactions.

Self assessment questions:

1. Define nuclear reaction . Discuss various types of nuclear reactions.
2. Explain the conservation laws that are obeyed by nuclear reactions.
3. What do you mean by nuclear kinematics? Obtain an equation for Q-value of the nuclear reaction.Explain the threshold energy of nuclear reaction.
4. Explain the principle and working of a nuclear reactor.
5. Discuss about the general features of nuclear reactor.
6. What are the various types of reactors? Explain them briefly.

Text books

1. Nuclear physics by D.C.Tayal , Himalaya publishing company,Bombay.
2. Nuclear physics by R.C.Sharma, K.Nath&co, Merut
3. Nuclear physics by S.B.Patel.

Unit 2

Lesson 5

NUCLEAR MODELS

The objectives of the lesson are to explain the following:

5.1 .Introduction

5.2.Shell model

5.3.Magic numbers

5.4.Extreme Single Particle Model

5.5.Semi Empirical Mass Formula

5.6.Liquid Drop Model

5.7.Summary

5.1. Introduction: It has been established that neutron and proton are the particles of which a nucleus is composed. The various properties, e.g., nuclear radius, nuclear mass, binding energy, packing fraction, magnetic moment, quadrupole moment etc. have also been dealt in detail in the same chapter. Now with the detailed study of nuclear forces it should be possible to interpret theoretically all the observed nuclear properties. Unfortunately, it has not been possible because two-body treatment can not be successfully applied to many body system because of much mathematical complexity . The total number of sub-nuclear particles is not very large, therefore, the statistical method to treat the problem becomes unsuitable and in the absence of any strong centre of force, the perturbation method can also not be applied. Thus, in the absence of suitable theoretical tool of investigation, physicists started a search for an alternative method of attack on the problem. This alternative method consists of looking round for a physical system - the model, the properties of which are known and they in turn are analogous to the properties of nucleus. In this way, on the basis of assumed models, physicists have tried to correlate various experimental data and to calculate some of properties.

A good model should describe

1. The properties of the ground nuclear states (spins, parities, magnetic dipole, magnetic dipole, electric quadrupole moment's etc.)

2.The properties of excited states especially the nuclear excitation spectrum.

3.The dynamical properties of the nucleus.

Ex. Probabilities of γ – quanta emission by individual nuclear excitation levels.

Physicists have restored to great number of nuclear models. No single model has provided a comprehensive description of the nucleus.

Nuclear models are various types and these are specified below.

1. Fermi gas model
1. Alpha particle model
2. Liquid drop model
3. Shell model
4. Collective model
5. Optical model

5.2.SHELL MODEL: Shell model can predict the magic numbers magnetic moment, quadrupole moment, ground state spin., and ground state energy. Evidences for the existence of shell model are given below.

5.3.Magic Numbers: The 1917, it was pointed out by Harkiins that nuclei with even numbers of protons or neutrons are more stable than those with odd numbers. Elasser, in 1934 observed that nuclides , which have either proton or neutron, number equal to 2,8,,20,50,82,126 shows pronounced stability. The numbers are called magic *numbers,* for the existence of such nuclides was not well understood for a long time. The name given is not scientific one but it serves the purpose of identifying the numbers conveniently. It was also observed that at these numbers nuclear properties show remarkable change. The nuclei such as O^{16} (Z = 8, N=8) and Pb^{208} (Z=82, N=126) are found exceptionally stable and are called doubly magic nuclei as both Z and N are magic numbers. It is further observed that as numbers 14,28 and 40, the stability is less than that for the above mentioned magic numbers and due to this reason the numbers are referred to as semi magic numbers.

Evidence for the existence of Magic numbers:

1. Mayer in 1948 suggested that nuclei with a magic number of nucleons are especially abundant in nature. Magic numbers are 2.,8,20,50,82, and126

2. ${}_2He^4$ and ${}_8O^{16}$ are particularly stable and it can be seen from the biding energy curve. The numbers 2 and 8 indicate stability because both are magic numbers. Further the stability of two nuclei is mainly due to consequence pairing of two protons and two neutrons align with opposite spins.

3. Above Z = 28 , the only nuclides of even Z which have isotopic abundances exceeding 60% are ${}_{38}St^{88}$ (N= 50) ${}_{56}Ba^{138}$ (N= 82) ${}_{58}Ce^{140}$ (N= 82)

4. No more than five isotones occur in nature for any neutron N except N = 50, where there are six and N = 82 where there are seven neutron numbers of 82, 50, there fore indicate particular stability.

5. Sn (Z = 50) has ten stable isotopes , more than any other element, while Ca (Z = 20) has six isotopes. This indicates that elements Z = 50 and Z = 20 are more stable than usually stable.

6. Alpha decay energies are rather smooth functions of A for a given Z but show striking discontinuities at N = 126 This represents the magic character of the number 126 for neutrons.

7. Very similar relations exist among the energies of beta ray emission. These energies are abnormally large when the neutron or proton of the final nucleus assumes a magic value.

8. The particularly weak binding of the first nucleon outside a closed shell is shown by unusually low probabilities for the capture of neutrons by nuclides having N = 50, 80 and 126

9. It is found that some isotones are spontaneous neutron emitters when excited above the nucleon binding energy by a preceding B- decay these are O (N= 9), Kr (N=51) Xe (N=83)

10. Nuclei with the magic proton numbers 50 (Sn) and 82 (Pb) have much smaller capture cross-sections than their neighbours.

11. The doubly magic nuclei (Z and N both magic numbers) $_2He^4$,$_8O^{16}$, $_{20}Ca^{40}$, $_{82}Pb^{208}$ are particularly tightly bound.

12. The binding energy of the next neutron or proton after a magic number is very small.

13. The asymmetry of the fission of uranium could involve. The sub-structure of nuclei, which is expressed in the existence of the magic numbers.

14. The Schmidt theory of magnetic moments for odd A nuclides show the ground states of these nuclides change from even parity to odd parity or vice versa at the numbers A = 4, 16, 40 when the nucleon numbers are 2, 8 and 20 respectively.

15. The electric quadrupole moment of nuclei show sharp minima at the closed shell numbers indicating that such nuclei are nearly spherical.

5.4)EXTREME SINGLE PARTICLE MODEL:

1) In this model it is assumed that the nucleons in the nucleus move independently in a common potential, determined by the average motion of all other nucleons.

2). Most of the nucleons are paired so that a pair of nucleons contributes zero spin and zero magnetic moment . The paired nucleus thus form an inert core.

3). The properties of odd A nuclei are characterized by the unpaired nucleon and odd – odd nuclei by the unpaired proton and neutron.

4).In order to understand some of the properties of nuclei including the magic numbers two cases are given.

a) Infinite square well potential

b) Harmonic oscillator potential

The square well potential has an infinitely sharp edge where as the harmonic oscillator potential diminishes steadily at the edge. The addition of spin orbit potential eliminates some of the difficulties experienced with the above two potentials.

Schrodinger wave equation for a particle moving in a spherically symmetric al central field of force. The eigen states available to a nucleon of mass M moving in a spherically symmetric potential V(r) determined by the solutions of equation.

$$\left(\nabla^2 + \frac{2M}{\hbar^2}\left(E - V(r)\right)\right)\psi(r) = 0$$

where E is the energy eigen value. Here reduced mass ‘μ’ is replaced by M which is equal to nucleon mass. The general solution of this equation can be written as

$$\psi_{n,l,m}(r,\theta,\Phi) = u_{n,l}(r)\ Y_{l,m}(\theta,\Phi)$$

When $u_{n,l}(r)$ is the radial function. $Y_{l,m}(\theta,\Phi)$ are the spherical harmonics. The set of quantum numbers n,l,m determines an eigen state corresponding to an eigen value $E_{n,l}$. The radial wave function $u_{n,l}(r)$ is solution of the equation.

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{du_{n,l}}{dr}\right)+\frac{2M}{\hbar^2}\left[E_{n,l}-V(r)-\frac{l(l+1)\hbar^2}{2Mr^2}\right]u_{n,l}=0$$

a) SQUARE WELL OF INFINITE DEPTH: Here we can calculate the position of various energy levels in an infinitely deep square well of radius 'R'. Let us assume that the potential is zero inside the well and infinite outside. Outside and at the boundary of well the radial wave function $u_{n,l}(r)$ vanishes. The radial wave functions are regular at the origin and inside the well are the spherical Bessel functions $J_l(K_{n,l}(r))$.

$$u_{n,l}(r)=j_l(k_{n,l}(r))=\sqrt{\frac{k}{r}}\,j_{l+1/2}(k_{n,l}(r))$$

where $j_{l+\frac{1}{2}}$ ($K_{n,l}$ (r)) is a Bessel function and $K_{n,I}$ is the wave number can be defined by the equation.

$$K_{n,l}{}^2=\frac{2M}{\hbar}^2\left(E_{n,l}-V(r)\right)$$

Where $E_{n,l}$ is the total – ve energy is the well depth . Energies are to be measured from the bottom of the well and then we have

$$V(r) = -V_0$$

$$K_{n,l}{}^2=\frac{2M}{\hbar^2}\left(E_{n,l}+V_0\right)$$

$$K_{n,l}{}^2=\frac{2M}{\hbar^2}E^1{}_{n,l} \qquad (\because E^1{}_{n,l}=E_{n,l}+V_0)$$

where $E^1{}_{n,l}$ is +ve , measured from the bottom of the well.

With the helpl of boundary conditions the permitted values of $k_{n,l}$ are selected. In the sample case or well or infinite depth, the wave function has to vanish at the nuclear boundary i.e., at r = R

$$u_{n,l}(R) = j_l\left(k_{n,l}(R)\right) = 0$$

Each l value has set of zeros and each of them corresponds to a value of $k_{n,l}$ given by $k_{n,l} = x$.

Thus the eigen value $K_{n,l}R$ is the n^{th} zero of the l^{th} spherical Bessel function . The numbers n, giving the number of zeros of the radial part of the wave function is known as radial quantum number . It differs from the principle quantum number of atomic spectroscopy. A graph of spherical Bessel function for $l = 0,1$ *and* 2 is shown in fig 5.1 . The order of a level in a spherical square well of infinite depth is given by the order of increasing energy. The energy levels are, therefore,

$$E^1_{n,l} = \frac{k^2_{n,l}\hbar^2}{2M} = \frac{\hbar^2}{2MR^2}\left(k_{n,l}R\right)^2 = \frac{\hbar^2}{2MR^2}x^2 \qquad (\text{since } k_{n,l}R = x)$$

Each shell model actually consists of $2l+1$ sub states. Each level has $2(2l+1)$ protons and $2(2l+1)$ neutrons.

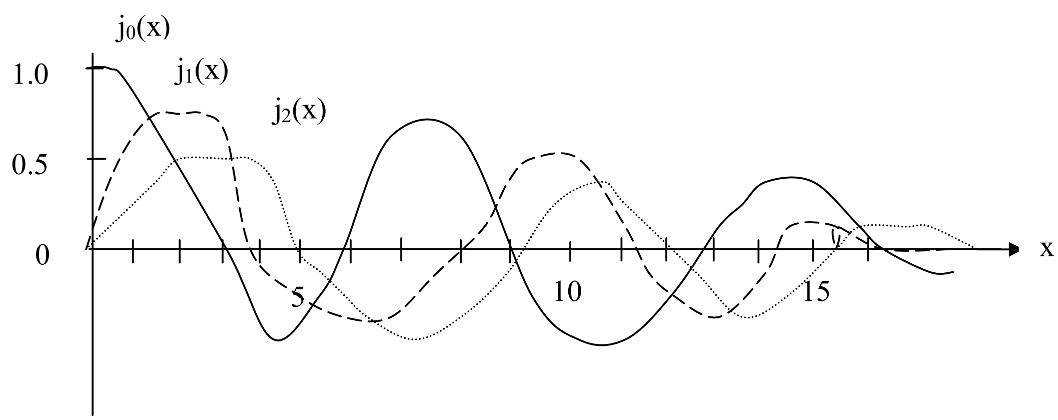


Fig.5.1 Spherical Bessel functions for $l = 0,1,2$

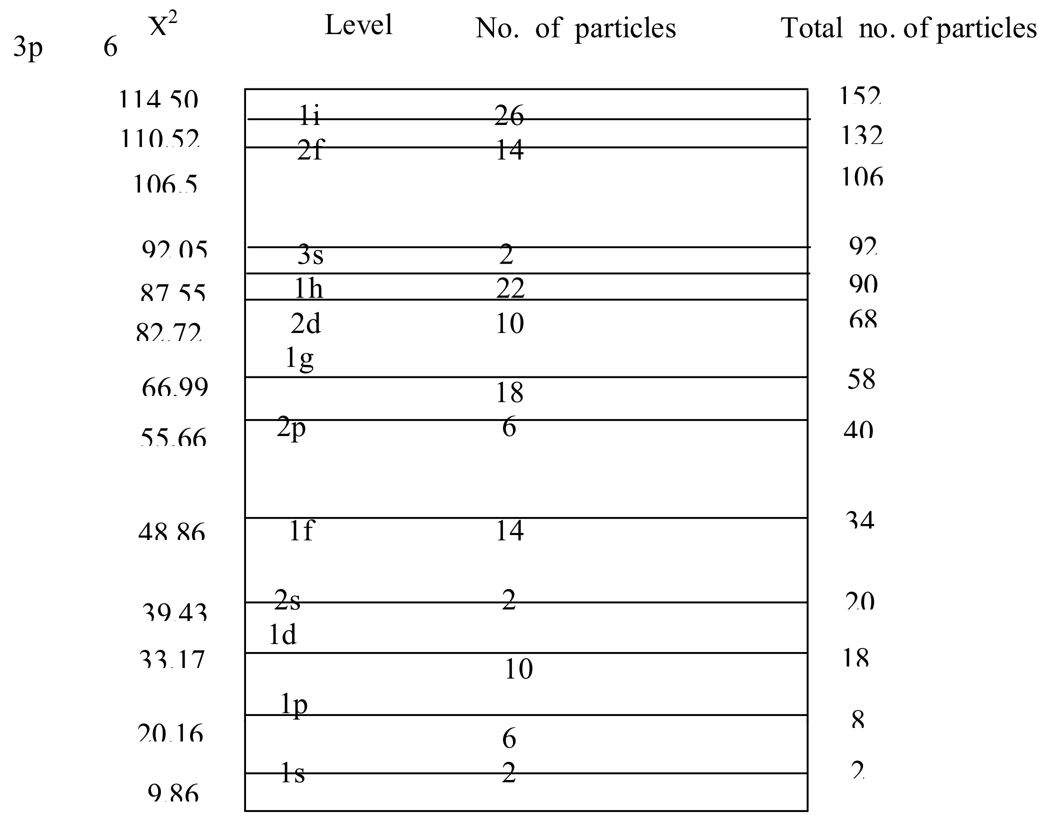


Fig:5.2 Levels and magic numbers predicted from infinitely deep square well.

Fig. 5.2 shows that the first energy level corresponds to $x = 3.14$ where $j_0(x) = 0$. *This* $l = 0$ state is written as 1s. It consists 2(2.0 +1) = 2 particles. The next level corresponds to $x = 4.49$, where $j_1(x) = 0$. This $l = 1$ state is written as 1p. It consists 2(2.1 +1) = 6 particles, thus giving altogether a total of eight other levels in

sequence are

$1d(l = 2, x = 5.76,$ *no.* of nucleons 10), 2s($l = 0$, $x = 6.28$, *number* 2),
$1f(l = 3, x = 6.99,$ *number* 14),......

These are listed in fig 5.2 . The left hand side represents the number of particles upto any particular level. Letter before terms s,p,d,f…. represents the order of zero in the Bessel function, e.g. 1s for first zero in Bessel function $j_l(x)$, 2s for second zero in this function .Here we see this shell closes at total particles 2,8,18,20,34,40,58,….. These are not the nuclear magic numbers.

b)Harmonic Oscillator potential: Consider a particle of mass 'M' moving with simple harmonic motion isotropically, bound in three dimensions held to a fixed point by a restoring force Kr, will vibrate about this fixed point with frequency

$$\upsilon = \frac{1}{2\pi}\sqrt{\frac{k}{M}} \quad or \quad \text{angular} \quad \text{velocity} \quad \omega = \sqrt{\frac{k}{M}}$$

Hence the P.E function V(r) of the oscillating particle

$$V(r) = \int_0^r kr\ dr = \frac{1}{2}kr^2 = \frac{1}{2}M\omega^2 r^2$$

The energy eigen value corresponding to the eigen function

$\psi_{n,l,m}(r,\theta,\Phi) = u_{n,l}(r)\ Y_{l,m}(\theta,\Phi)$ is given by

$$E_{n,l} = \hbar\omega(2n + l - \frac{1}{2}) = \left(\Lambda + \frac{3}{2}\right)\hbar\omega$$

$$n = 1,2,3,\ldots\ldots\ldots\ldots\ldots \qquad l = 0,1,2,\ldots\ldots\ldots\ldots \qquad \Lambda = 2n + l - 2$$

When the angular dependence of the wave function is examined, it is found that for each Λ value , there is a degenerate group of levels with different l values such that $l \le \Lambda$ and even (odd) corresponds to even(odd) values of Λ. Thus the sequence of single degenerate levels, each band separated by energy $\hbar\omega$ from the next.. The degeneracy corresponding to each l value is $2(2l+1)$ as before.

However, the eigen states corresponding to the same value of Λ (=2n +l-2) are also degenerate . Since 2n = Λ - l + 2 is even for Λ even or odd, the degenerate eigen states are

$$(n,l)=\left[\frac{1}{2}(\Lambda+2),0\right],\left[\frac{\Lambda}{2},2\right]....................[2,\Lambda-2],[1,\Lambda] \quad for \quad \Lambda \quad even$$

$$=\left[\frac{1}{2}(\Lambda+1),1\right],\left[\frac{1}{2}(\Lambda-1),3\right],.............[2,\Lambda-2],[1,\Lambda] \quad for \quad \Lambda \quad odd$$

No. of neutrons or protons with the eigen value E_Λ

$$N_A=\sum_{k=0}^{\Lambda/2}2[2(2k)+1] \qquad for \quad even \quad \Lambda$$

$$=\int_{k=0}^{\frac{\lambda-1}{2}}2[2(2k+1)+1] \qquad for \quad odd \quad \Lambda$$

$$=(\Lambda+1)(\Lambda+2) \text{ in either case}$$

The total no. of particles for all levels up to Λ is

$$\sum_{\Lambda}N_\Lambda=\frac{1}{3}(\Lambda+1)(\Lambda+2)(\Lambda+3)$$

The single particle level scheme predicted by the infinite harmonic oscillator well is given below.

Λ	E_Λ	l values	Degenerate states	No. of particles $(\Lambda+1)(\Lambda+2)$	Total no. of particles $\frac{1}{3}(\Lambda+1)(\Lambda+2)(\Lambda+3)$
0	$\frac{3}{2}\hbar\omega$	0	1s	2	2
1	$\frac{5}{2}\hbar\omega$	1	1p	6	8
2	$\frac{7}{2}\hbar\omega$	0,2	2s,1d	12	20
3	$\frac{9}{2}\hbar\omega$	1,3	2p,2f	20	40
4	$\frac{11}{2}\hbar\omega$	0,2,4	3s,2d,1g	30	70

The result of harmonic oscillator potential indicates the shell closes at the numbers 2,8,20,40,70 etc, where as square well potential suggests the magic numbers at 2,8,20,34,40,58,68,70,92,106,112,138 and 156. Experimentally observed values are 2,8,20,50,82,126. There fore, the truth lies between these two potential.

C)Spin Orbit Coupling: Mayer and Hexel, Jensen and suess in 1949 suggested that a non-central component should be included in the force acting on a nucleon in a nucleus. Spin orbit coupling is corresponding to the interaction between the orbital angular momentum and the intrinsic angular moment (spin) of a particle. The magnetic moment is associated with the spin angular momentum and the magnetic field is induced due to the orbital angular momentum . This magnetic field has an effect on the magnetic moment.

The interaction energy
$$W = -\vec{\mu}_s.\vec{B}$$
$$= \quad -f(r).\hat{s}.\hat{l}$$

Here μ_s is the magnetic moment, B is the magnetic field.

S and l are the spin and orbital angular momentum vectors respectively.

f (r) is the potential function. The potential describing the single particle wave function will be $V(r) - f(r)\hat{s}.\hat{l}$ V(r) is the central potential, f (r) is non-central potential component. They are dependent only on the radial distance and the size of the nucleus. Because of the strong coupling the two vectors combine to a total angular momentum j for this particle since s = ½ there are only two possible ways of s and l , resulting in the Stretch case j = l + s and Jacknife case j = l - s (see fig. 5.3)

$$s,l = \frac{1}{2}l \quad for \quad j = l + \frac{1}{2}$$
$$s,l = -\frac{1}{2}(l+1) \quad for \quad j = l - \frac{1}{2}$$

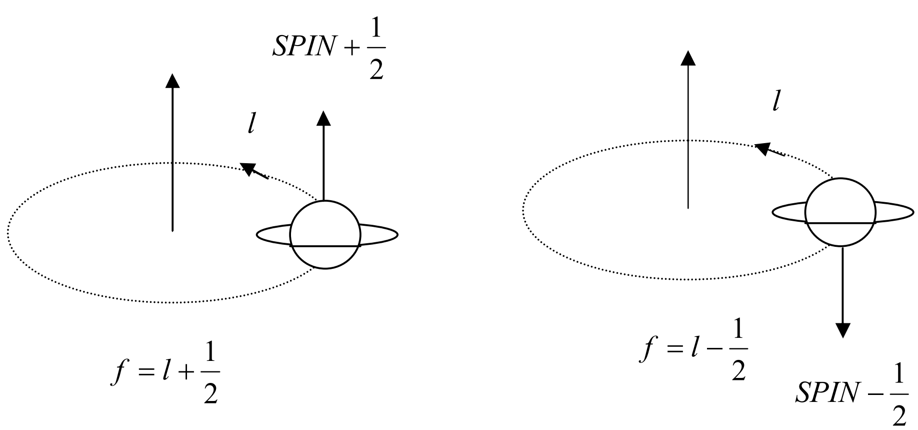


Fig.5.3.Coupling of orbital and spin angular momenta of a nucleon.

The spin orbit interaction splits each of the higher single particle levels. For a given l and they are characterized by total angular momentum quantum numbers $l+\frac{1}{2}, l-\frac{1}{2}$ and each sublevel can accommodate (2j+1) neutrons and protons.

Splitting of nucleon energy levels based on shell model.

The inherent assumptions of a single particle shell model are listed below.

1. The levels are gradually filled, the levels with higher j lies deepest.

2. The energy difference between $l+\frac{1}{2}$ level and $l-\frac{1}{2}$ level is quite large for a given l and it increases with increasing l

3. An even no. of identical nucleon with same l and j couple to keep even parity (zero total angular momentum and zero magnetic moment.)

4. An odd no. of identical nucleon with same l and j couple to give rise odd parity. if l is odd and even parity if l is even and total angular momentum j and magnetic moment is equal to that of single nucleon in that state.

5. If the state is occupied by two identical nucleons, there is additional paring energy associated with the states and the pairing energy increasing with j value.

On the basis of above assumption the existence of higher magic numbers can be deduced . As an example consider the 1d state. The state orbital angular momentum of 2 units and spin angular momentum of ½ unit; hence there are two possible values of total angular momentum 5/2 and 3/2 . the former is six fold degenerate and latter is four fold; on the whole 1d state, can accommodate 10 nucleons. When the value of l becomes large, the separation of $l-\frac{1}{2}$ level with $l+\frac{1}{2}$ level increases to such an extent that two belong to the different shells.

The situation is illustrated in figure5.4. for the levels 1g $_{9/2}$ (N=4), 1h$_{11/2}$ (N=5),etc. In figure 5.4, 2 digits on the right side give the magic numbers. The energy values are mentioned on the left side after subtracting zero point energy.

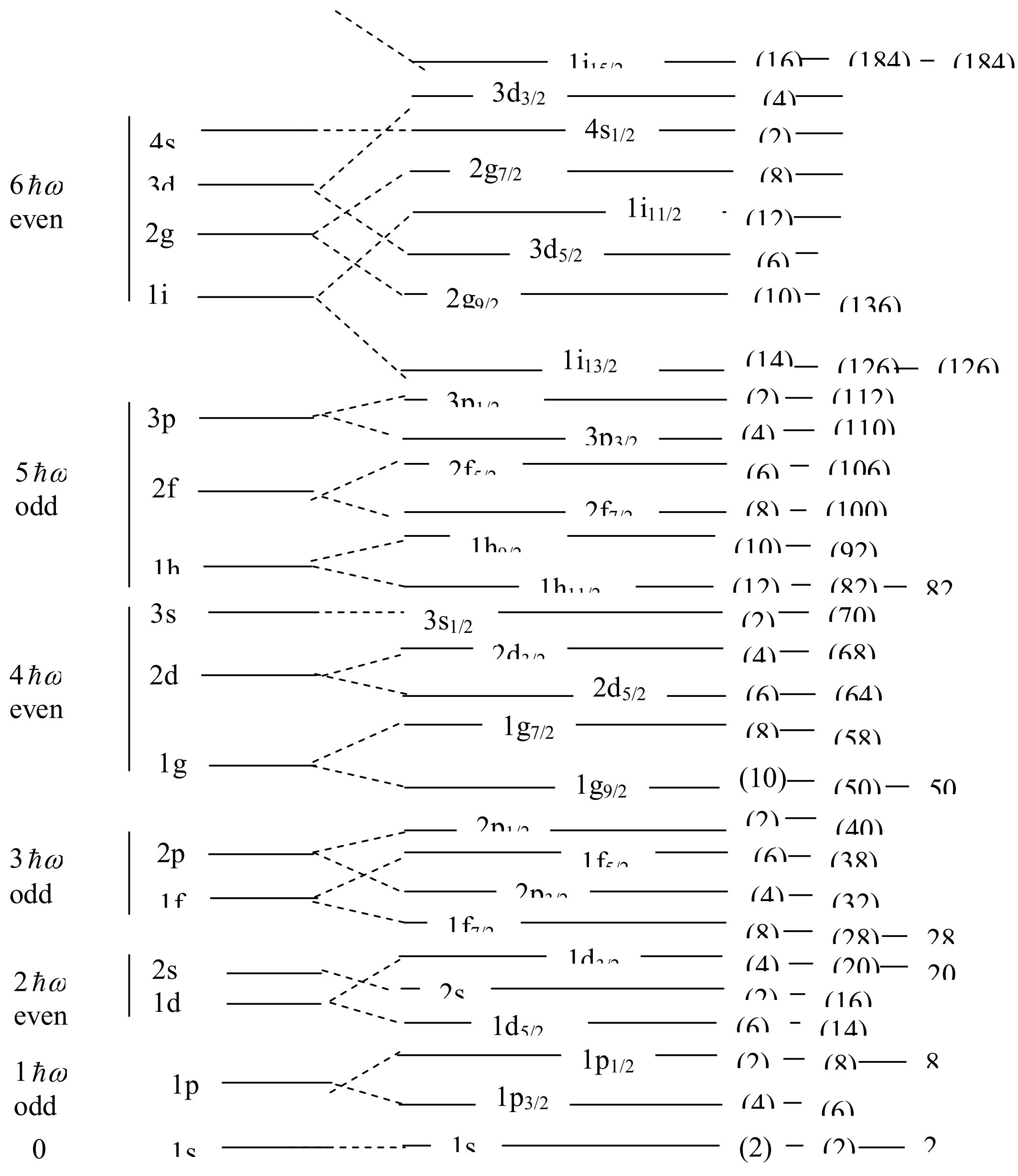


Fig. 5.4. splitting of energy levels in shell model

Examples of finding spin & parity of elements.

1) $_6C^{12}$ 6p & 6N I = 0 parity = $(-1)^0$ = +1 even

2) $_5B^{11}$ 5p & 6N $I = \frac{3}{2}$ parity = $(-1)^1$ = -1 odd

3) $_7N^{16}$ 7p & 9N $J_1 = \frac{1}{2}, l_1 = 1$ $J_1 + J_2 + l_1 + l_2 = 6$ *even*

$J_2 = \frac{5}{2}, l_2 = 1$ $I = \left(\frac{1}{2} - \frac{5}{2}\right) = 2$

$P = (-1)^{l_1 + l_2} = (-1)^3 = -1$ *odd*

4) $_{33}Ar^{73}$ 33p & 42N $I = \frac{3}{2}$ P = $(-1)^3$ = -1 odd

5) $_{28}Ni^{61}$ 28p & 33N $I = \frac{3}{2}$ P = $(-1)^3$ = -1 odd

4)Predictions of the shell model:

1)Stability of the closed shell nuclei: This scheme clearly reproduces all the magic numbers.

2)Spins and parities of nuclear ground states:

According to shell model the neutron and protons levels fill independently. There are following rules for the angular momenta and parities of nuclear ground state.

a)Even-even nuclei have total ground state angular momentum I = 0 there is no known exception to this rule.

b) With an odd number of nucleons i.e. a nucleus with odd z or odd N the nucleons pair off as far as possible so that the resulting orbital angular momentum and spin direction are just that of single odd particle. There are some exception to this rule and they will be discussed below.

c)An odd-odd nucleus will have total angular momentum which is the vector sum of the odd neutron and odd proton j values. The parity will be the product of the proton & neutron parity i.e..

Parity $=(-1)^{l_n+l_p}$

In the case of odd-odd nuclei the total angular momentum is due to the last proton and last neutron which are left as unpaired neutrons in the respective quantum state. The angular momentum of these two nucleons can combine in many ways. And resultant angular momentum is found by "Nordhiem" rule which states that if for two odd nucleons $j_1 + j_2 + l_1 + l_2$ is an even no. the resultant angular momentum I = $|J_1 - J_2|$

If $j_1 + j_2 + l_1 + l_2$ is an odd no., the spin I is large approaching I = $J_1 + J_2$

For the determination of resultant angular momentum of odd-odd nuclei we apply "Nordheim" rule

If $j_1 + j_2 + l_1 + l_2$ = even no., Spin I = $|J_1 - J_2|$

If $j_1 + j_2 + l_1 + l_2$ = odd no., Spin I = $J_1 + J_2$

5.5.SEMIEMPIRICAL MASS FORMULA:

The mass M of a neutral atom whose nucleus contains Z protons and A-Z neutrons is given by

$$_{Z}M^{A} = ZM_P + (A-Z)M_N + ZM_e - E_B$$

$$= ZM_H + (A-Z)M_N - E_B$$

Where E_B is the binding energy . may be made up of number of terms, each of which represents some general characteristics of nuclei

$$E_B = E_v + E_s + E_c + E_a + E_p$$

Where E_v is volume binding energy, E_s is surface binding energy,E_c is coulomb binding energy. E_a is Asymmetric binding energy. E_p is pairing binding energy

Von weizsacker obtained the above formula in 1935. $_ZM^A$ is called semi-empirical mass formula.

The semi-empirical mass formula possesses following properties.

1.It explains the stability of the nuclide

2.it explains fission of heavy nuclide

3.it explains some of the regularities associated with alpha decay process

4.This formula is based upon the general structure of the nucleus and the nature of the binding forces i.e., it incorporate the following assumptions.

a) Nucleus is a droplet of incompressible matter (nuclear density is 10^{14} g/cc)

b)Nuclear forces are short range forces and possess saturation property.

c)There are electrostatic repulsion forces and surface tensional effects that will exists in nuclei.

1)Volume energy: The neutrons and protons are held together by short range attractive forces. These forces reduces the mass of the nucleus below that of its constituents by an amount proportional to the number of nucleons A since the volume of the nucleus is proportional to A hence this term may be regarded as a volume binding energy often called exchange energy and is given by E_v

$$E_v \; \alpha \; A \;\Rightarrow\; E_v = a_v A$$

Where a_v is proportionality constant and subscript v is volume energy.

2) Binding Energy: Since nulceus is finite, some nucleons are nearer to the surface so that they interact with few nucleons thus the binding energy E_v is reduced by an amount proportional to the surface area of the nucleus of radius R as the nucleons on the surface are less tightly bound than those in the interior. For light nuclei nearly all the nucleons are at the surface while for heavy nuclei about half the nucleons are at the surface. Thus the surface energy is analogous to the surface tension of nucleus.

$$E_s \ \alpha \ 4\Pi R^2$$

$$E_s \ \alpha \ 4\Pi(R_0A^{1/3})^2 \quad \text{R is radius of spherical nucleus}$$

$$E_s = {}_{-}a_s A^{2/3}$$

Where a_s is proportionality constant and subscript means surface energy.

3)Coulomb energy: Long range force in the nuclei is the coulomb repulsion between protons. The coulomb repulsion term is equal to potential energy of Z protons packed together in a spherically assembly of mean radius R= $R_0A^{1/3}$

$$\text{Uniform charge density } \rho_c = \frac{Ze}{\frac{4}{3}\pi R_0{}^3 A}$$

The electrostatic energy is simply the work done against electrostatic forces in assembling such a sphere. Let dq be the charge on the shell of thickness dr on the sphere of radius 'r'

$$dq = \rho_c \times 4\pi r^2 dr$$

The work done to bring the charge from infinity, to 'r' against the charge on the sphere.

$$dq = \frac{4\pi r^2 \rho_c \frac{4}{3}\pi r^3 \rho_c}{r}\frac{1}{4\pi\varepsilon_0}$$

$$E_c = -\int_0^R \frac{16\pi^2 r^5 \rho^2 dr}{3r}\frac{1}{4\pi\varepsilon_0}$$

$$= -\int_0^R \frac{16\pi^2 r^4 \rho^2 dr}{3}\frac{1}{4\pi\varepsilon_0}$$

$$= \frac{-16\pi^2 R^5 \rho^2}{15}\frac{1}{4\pi\varepsilon_0}$$

$$E_c = \frac{-3z^2e^2}{5R}\frac{1}{4\pi\varepsilon_0} \qquad (\sin ce\ \rho_c = \frac{Ze}{\frac{4}{3}\pi R^3})$$

$$E_c = \frac{-3z^2e^2}{5\pi R_0 A^{\frac{1}{3}}}\frac{1}{4\pi\varepsilon_0}$$

Coulomb repulsion binding energy $E_c = -\frac{3}{5}\frac{z^2e^2}{4\pi\varepsilon_0 R_0 A^{1/3}} = -a_c\frac{Z^2}{A^{1/3}}$

where a_c is a constant and the subscript c denotes coulomb energy.

4)Asymmetry energy: It has been observed that nuclei are most stable when nucleus consists of n equal number of protons and neutrons . As the value of A increases the number of protons increases and this transition to the asymmetric configuration from symmetric configuration reduces the stability of nuclei. Alternately, the binding energy decreases and this decrement is known as correction. (N –Z) excess of neutrons produces a deficit in binding energy because they are out of reach of other nucleon. The fraction of nuclear volume so affected is $\frac{N-Z}{A}$ and there fore, the total deficit should be proportional to

$$(N-Z)\frac{(N-Z)}{A} \quad or \quad \frac{(A-2Z)^2}{A}$$

$$hence \quad B_3 = -a_a\frac{(A-2Z)^2}{A}$$

In fact asymmetry energy is purely quantum mechanical effect in contrast with coulomb energy and surface energy. The presence of unbinding term $(N-Z)^2$ greatly favours Z =N, as stable configuration.

$$E_a \alpha \quad (N-Z)\left(\frac{N-Z}{A}\right)$$

$$E_a = -a_a\frac{(N-Z)^2}{A}$$

$$E_a = -a_a\frac{(A-2Z)^2}{A}$$

a_a is constant subscript represents 'a ' is for asymmetry.

5) Pairing energy: The nuclides with even numbers of protons and neutrons are the most abundant and most stable. Nuclei with odd numbers of both neutrons and protons are the least stable. While nuclei for which either proton or neutron number is odd are intermediate in stability. To take account of this pairing effect an additional term is used. This term can be taken as zero for odd A, $-\delta$ for odd-odd and $+\delta$ for even-even nuclide.

$\delta = ap\,A^{-\frac{3}{4}}$ where a_p is the pairing energy constant.

$E_p = 0$ for A odd, Z even N odd or Z odd N even

$E_p = a_p/A^{3/4}$ for A even, Z even and N even

$E_p = -a_p/A^{3/4}$ for A even, z odd and N odd.

By collecting together the different stability of mass correct terms, then semi empirical mass formula becomes.

$$_ZM^A = ZM_H + (A-Z)M_N - a_vA + a_sA^{2/3} + a_cZ^2 A^{-1/3} + a_a (A-2Z)^2 A^{-1} \pm a_pA^{-3/4}$$

This is called semi empirical mass formula.

Applications:

1) Fission of heavy nuclides
2) Behaviour of isobars in B –decay
3) Alpha decay , coulomb radius, radius of mirror etc, are excellent agreement with other methods.

Modifications:

$$_ZM^A = Z(M_H - M_N) + A(M_N - a_v) + a_sA^{2/3} + a_cZ^2 A^{-1/3} + a_a A - 4 a_a Z + 4 a_a Z^2/A \mp \delta$$

$$_ZM^A = a_1A + a_2Z + a_3Z^2 \mp \delta \text{ …………….(1)}$$

Where $a_1 = M_N - (a_v - a_a - a_sA^{-1/3})$

$a_2 = M_H - M_N - \psi a_a$

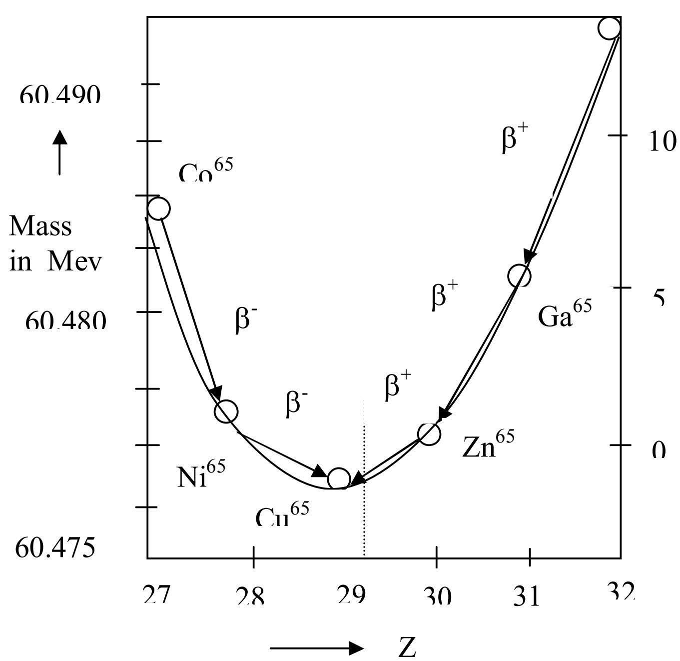
$$a_3 = 4\, a_a/A + a_c/A^{1/3}$$

eqn(1) gives the dependence of nuclear mass on nuclear charge for a constant mass number 'A'. This dependence gives parabola. Therefore it is very clear that most stable nuclide has the minimum mass corresponding value of 'Z' can be determined by finding the minimum of parabolic curve. Accordingly differentiate eqn(1) w.r.to 'Z' for constant A then we get

$$\frac{\partial\left({}_zM^A\right)}{\partial z} = a_2 + 2a_3 z = 0$$

$$\Rightarrow z_0 = -\frac{a_2}{2a_3}$$

This gives the value of 'z' for most stable isobar for given 'A'.



Schematic plote of M versus Z for odd A

5.6.LIQUID DROP MODEL:

In 1937, N. Bohr proposed the liquid drop model for the nucleus associated with semi empirical mass formula . In this model the finer features of nuclear forces are ignored but the strong inter nucleon attraction is stressed. The essential assumptions are

1) The nucleus consists of incompressible matter so that $R = R_0A^{1/3}$

2) The nuclear force is identical for every nucleon.

3) The nuclear force saturates.

The individual nucleons must be able to move about with in the nucleus much as does an atom of a liquid, therefore think of nucleus as being like a small drop of liquid such a model is known as liquid drop model.

The idea that the molecules in the drop of a liquid corresponding to the nucleons in the nucleus is confirmed due to the following similarities.

1) The nuclear forces are analogous to the surface tension force of a liquid.

2) The nucleons behave in a manner similar to that of molecules in a liquid.

3) The density of nuclear matter is almost independent of mass number 'A' which is analogous to the liquid drop where the density of liquid is independent of the size of drop.

4) The constant binding energy per nucleon is analogous to the latent heat of vapourisation

5) The disentegration of nuclei by the emission of particles is analogous to the evaporisation of molecules from the surface of liquid.

6) The energy of nuclei corresponds to international thermal vibrations of drop molecules.

7) The formation of compound nucleus and absorption of bombarding particles are correspond to the condensation of drops.

Inspite of these similarities we see some of the following differences.

1) Molecules attract one another at distances larger than the dimensions of the electron shells and repel strongly when the distance is smaller than the size of the electron orbits. Nuclear forces are attractive within the smaller range, the range of nuclear forces.

2) The average K.E. of molecule in the liquid is of the order of 0.1ev., the corresponding de Broglie wave length is 5×10^{-11}m. which is very much smaller than the inter nuclear distances The average K.E. of nucleons in a nuclei is of the order of 10Mev corresponding to wavelength is 6×10^{-15}m, which is the order of inter – nucleon distances. Hence the motion of molecules in the liquid is of classical character where as in nuclei the motion of the nucleons is of quantum character.

The success of this model has been beautifully demonstrated in explaining the following

1) Constant binding energy per nucleon
2) Behaviour of isobars
3) Constant density of nucleons with radius $R = R_0A^{1/3}$
4) Fission heavy nuclei
5) Nuclear reaction
6) Cross-section for resonance reaction
7) Stability limit of heavy nuclei.

The condition for fission: Let us consider the case of symmetrical fission in order to simplify the calculation. The fission energy E_f or Q - value for the nuclear fission relation will be written as

$$E_f = B_A - 2B_{\frac{A}{2}}$$

In the spontaneous fission

$$E_f = {}_zM^A - 2\,{}_{\frac{z}{2}}M^{\frac{A}{2}}$$

Semi empirical mass formula is

$${}_zM^A = ZM_H + (A-Z)M_n - a_v A + a_c \frac{Z^2}{A^{1/3}} + a_s A^{2/3} + a_a \frac{(A-2Z)^2}{A}$$

$${}_{\frac{z}{2}}M^{\frac{A}{2}} = \frac{Z}{2}M_H + \left(\frac{A}{2}-\frac{Z}{2}\right)M_n - a_v \frac{A}{2} + a_c \frac{\left(\frac{Z}{2}\right)^2}{\left(\frac{A}{2}\right)^{1/3}} + a_s \left(\frac{A}{2}\right)^{2/3} + a_a \frac{\left(\frac{A}{2}-2\frac{Z}{2}\right)^2}{\frac{A}{2}}$$

$$E_f = {}_zM^A - 2\,{}_{\frac{z}{2}}M^{\frac{A}{2}}$$

$$E_f = ZM_H + (A-Z)M_n - a_v A + a_c \frac{Z^2}{A^{1/3}} + a_s A^{\frac{2}{3}} + a_a \frac{(A-2Z)^2}{A} -$$

$$2\left[\frac{Z}{2}M_H + \left(\frac{A}{2}-\frac{Z}{2}\right)M_n - a_v \frac{A}{2} + a_c \frac{\left(\frac{z}{2}\right)^2}{\left(\frac{A}{2}\right)^{1/3}} + a_s \left(\frac{A}{2}\right)^{2/3} + a_a \frac{\left(\frac{A}{2}-\frac{2Z}{2}\right)^2}{\frac{A}{2}}\right]$$

$$E_f = \underbrace{a_s\left[A^{\frac{2}{3}} - 2\left(\frac{A}{2}\right)^{2/3}\right]}_{Surface\ energy} + \underbrace{a_c\left[\frac{Z^2}{A^{1/3}} - 2\frac{(Z/2)^2}{(A/2)^{1/3}}\right]}_{Coulomb\ energy}$$

$$a_s = 13.1 Mev$$
$$a_c = 0.609 Mev$$

Spontaneous fission is energetically possible only if $E_f \geq 0,$ *then*

$$-3.42A^{2/3} + 0.22\frac{Z^2}{A^{1/3}} \geq 0$$

$$\frac{Z^2}{A} \geq \frac{342}{22}$$
$$\frac{Z^2}{A} \geq 15$$

This condition is satisfied with mass number A > 85

The fission process affects these two energies in opposite ways. The tendency of one is to annual partly other. It is expected in the fission process because in the division of the nucleus, the separation between the proton groups increases reducing the coulomb energy while the total nuclear surface increases, increasing the surface energy.

BOHR WHEELER'S THEORY OF NUCLEAR FISSION:

Nuclear distortion explained on the basis of liquid drop .

The nuclear distribution have an appreciable influence on the surface energy and coulomb energy. The nuclear distribution can be stated by assuming that the shape of the nucleus is not sphere. The small distribution of liquid drop is given by

$$r_\theta = R\, f(\theta)$$

r_θ is the radius of deformed drop

where f(θ) can be expressed as an expression in the Legendre's polynomial.

$$f(\theta) = 1 + a_0 p_0(Cos\theta) + a_1 p_1(Cos\theta) + a_2 p_2(Cos\theta) + ... + a_n p_n(Cos\theta)$$

$$r_\theta = R + R\sum_{n=0}^{\alpha} a_n p_n(Cos\theta) \qquad for \quad n = 0,1,2,3...........$$

In f(θ) where $a_0, a_1, a_2.....$ are the distortion parameters and p's are the Legendre's polynomial

$$p_0(\cos\theta) = 1 \qquad p_1(\cos\theta) = Cos\theta,$$
$$p_2(\cos\theta) = \frac{3Cos^2\theta - 1}{2}, \qquad p_3(\cos\theta) = \frac{5Cos^3\theta - 3Cos\theta}{2}$$

Here, the requirement of constant volume specifies that $a_0 = a_1 = 0$

Because the centre of mass of the drop is assumed to remain unchanged for these two cases.

$$f(\theta) = 1 + \sum_{l=0}^{\alpha} a_l p_l(Cos\theta) \text{.............}(1)$$

$$f(\theta) = 1 + \sum_{r=2}^{\alpha} a_r p_r(Cos\theta)$$

From orthogonality property of Legendre's polynomial the distortion parameter for f(θ) can be written as

$$a_n = \frac{\int_0^\pi f(\theta) p_n(Cos\theta) Sin\theta d\theta}{\int_0^\pi p_n^2(\cos\theta) Sin\theta d\theta} \text{...................}(2)$$

proof:

we know that

$$\int_{-1}^{+1} p_n^2(n) dn = \frac{2}{2n+1}$$

$$\int_{-1}^{+1} p_m(n) p_n(n) dn = 0 \qquad m \neq n$$

Multiplying (1) by $p_n(\cos\theta) Sin\theta d\theta$ and integrate between the limits 0 to π

$$\int_0^\pi f(\theta) p_n(\cos\theta) Sin\theta d\theta = \int_0^\pi 1\, p_n(Cos\theta) Sin\theta d\theta + \sum_{l=0}^{\alpha} a_l \int_0^\pi p_l(\cos\theta) p_n(Cos\theta) Sin\theta d\theta$$

$$\int_0^\pi P_0(\cos\theta) p_n(Cos\theta) Sin\theta d\theta + \sum_{l=0}^{\alpha} a_l \int_0^\pi p_l(\cos\theta) p_n(Cos\theta) Sin\theta d\theta$$

In the above equation l = n is the only non vanishing term hence we get i.e., $\int_0^\pi a_n p_n^2(Cos\theta) Sin\theta d\theta = \int_0^\pi f(\theta) p_n(Cos\theta) Sin\theta d\theta$

$$a_n = \frac{\int_0^{\pi} f(\theta) p_n (Cos\theta) Sin\theta d\theta}{\int_0^{\pi} p_n^{\,2} (\cos\theta) Sin\theta d\theta}$$

The small distribution of a liquid can be written as

$$r_\theta = Rf(\theta)$$

$$r_\theta = R[1 + a_2 p_2(\cos\theta) + a_3 p_3(Cos\theta) + ... + a_n p_n(Cos\theta)]$$

For a's which are identical with one another and equal to zeros nucleus is an undistorted sphere of radius $r_\theta = R$.

Surface energy of undistorted sphere is

$E_s^0 =$ surface area x surface tension coefficient

$$= 4\pi R^2 \times T$$

$$E_s^{\,o} = 4\pi R_0^{\,2} T A^{2/3} = a_s A^{2/3}$$

Total surface energy of the distorted nucleons is given by

$$E_s^{\,o} = 4\pi r_\theta^{\,2} T = 4\pi R^2 T[f(\theta)]^2$$

$$E_s^{\,o} = 4\pi R^2 T[1 + a_2 p_2(Cos\theta) + a_3 p_3(\cos\theta) + ...]^2$$

$$= 4\pi R_0^{\,2} A^{2/3} T\left[1 + a_2\left(\frac{3Cos^2\theta - 1}{2}\right) + a_3\left(\frac{5Cos^3\theta - 3Cos\theta}{2}\right) + ...\right]^2$$

$$E_s^{\,o} = 4\pi R_0^{\,2} A^{2/3} T\left[1 + a_2^{\,2}\frac{2}{5} + a_3^{\,3}\frac{5}{7} + ...\right]$$

By neglecting higher order terms

Surface energy of deformed nucleus is

$$E_s^{\,o} = 4\pi R_0^{\,2} T A^{2/3}\left[1 + \frac{2}{5}a_2^{\,2}\right] ....(3)$$

$$E_s^{\,o} = a_s A^{2/3}\left[1 + \frac{2}{5}a_2^{\,2}\right] \qquad \left[\because a_s = 4\pi R_0^{\,2} T\right]$$

Electrostatic energy or coulomb energy of undistorted spherical shape is

$$E_c^{o} = \frac{3}{5}\frac{Z^2e^2}{R} = \frac{3}{5}\frac{Z^2e^2}{R_0A^{1/3}} = a_c\frac{Z^2}{A^{1/3}} \qquad where \qquad a_c = \frac{3}{5}\frac{e^2}{R_0}$$

similarly, the coulomb energy of deformed nucleus as

$$E_c^{o} = \frac{3}{5}\frac{Z^2e^2}{r_\theta} = \frac{3}{5}\frac{Z^2e^2}{Rf(\theta)}$$

$$= \frac{3}{5}\frac{Z^2e^2}{R_0A^{1/3}}\left[1 + a_2\left(\frac{3Cos^2\theta - 1}{2}\right) + a_3\left(\frac{5Cos^3\theta - 3Cos\theta}{2}\right) + ...\right]^{-1}$$

$$= \frac{3}{5}\frac{Z^2e^2}{R_0A^{1/3}}\left[1 - \frac{1}{5}a_2^{\,2} + \frac{10}{49}a_3^{\,2} + ...\right]$$

$$= \frac{3}{5}\frac{Z^2e^2}{R_0A^{1/3}}\left[1 - \frac{1}{5}a_2^{\,2}\right] \quad (By \text{ neglecting the higher order terms})$$

$$= a_c\frac{Z^2}{A^{1/3}}\left[1 - \frac{1}{5}a_2^{\,2}\right]$$

Total energy of undistorted nucleus

$$E_s^{\,0} + E_c^{\,o} = 4\pi R_0^{\,2}TA^{2/3} + \frac{3}{5}\frac{Z^2e^2}{R_0A^{1/3}}$$

$$= a_sA^{2/3} + a_c\frac{Z^2}{A^{1/3}}$$

Total energy of distorted nucleus

$$E_s^{\,0} + E_c^{\,o} = 4\pi R_0^{\,2}TA^{2/3}\left(1 + \frac{2}{5}a_2^{\,2}\right) + \frac{3}{5}\frac{Z^2e^2}{R_0A^{1/3}}\left(1 - \frac{1}{5}a_2^{\,2}\right)$$

$$= a_sA^{2/3}\left(1 + \frac{2}{5}a_2^{\,2}\right) + a_c\frac{Z^2}{A^{1/3}}\left(1 - \frac{1}{5}a_2^{\,2}\right) ....(7)$$

The total energy of distorted spherical shape which differ from former case because of the increase of surface energy due to increase in surface area and decrease of coulomb energy because of the centre of binary parts being further apart.

Now, distortion is given by

$$\Delta E_{s,c}{}^{0} = \left(E_s + E_c\right)^{o} - \left(E_s{}^{o} + E_c{}^{o}\right)$$

$$= a_s A^{2/3}\left(1 + \frac{2}{5}a_2{}^2\right) + a_c\frac{Z^2}{A^{1/3}}\left(1 - \frac{1}{5}a_2{}^2\right) - \left(a_s A^{2/3} + a_c A^{-1/3}\right)$$

$$= a_s A^{2/3}\frac{2}{5}a_2{}^2 - a_c\frac{Z^2}{A^{1/3}}\frac{1}{5}a_2{}^2$$

$$= \frac{1}{5}a_2{}^2\left(2a_s A^{2/3} - a_c\frac{Z^2}{A^{1/3}}\right)$$

$$= \frac{1}{5}a_2{}^2\left(2E_s{}^{o} - E_c{}^{0}\right)\text{.....................(8)}$$

For the above expression it is obvious that if $2E_s{}^0 \succ E_c{}^0$ then $\Delta E_{s,c}{}^0$ is +ve, the nucleus is metastable.

If $2E_s{}^0 \prec E_c{}^0$ then $\Delta E_{s,c}{}^0$ is -ve, then the nucleus is completely unstable

If $2E_s{}^0 = E_c{}^0$ then $\Delta E_{s,c}{}^0 = 0$ which design a critical value for $\frac{Z^2}{A}$.

From (8)

$$\Delta E_{s,c}{}^0 = \frac{1}{5}a_2{}^2\left(2E_s{}^0 - E_c{}^0\right)$$

$$= \frac{1}{5}a_2{}^2\left[2\left(4\pi R_0{}^2 T A^{2/3}\right) - \frac{3}{5}\frac{Z^2e^2}{R_0 A^{1/3}}\right]$$

If $2E_s{}^0 = E_c{}^0$ so that

$$2\left(4\pi R_0{}^2 T A^{2/3}\right) = \frac{3}{5}\frac{Z\ e^2}{R_0 A^{1/3}}\frac{1}{4\pi\varepsilon_0}\frac{Z^2}{A} = \frac{24\pi R_0{}^2 T 5 R_0 4\pi\varepsilon_0}{3e^2}$$

$$\left(\frac{Z^2}{A}\right)_{critical} = \frac{4\pi\varepsilon_0 40\pi R_0{}^3 T}{3e^2}$$

$$put \quad R_0 = 1.6F \quad = 1.6\times10^{-15}\, m$$

$$e = 4.8\times10^{-10}\, esu$$

$$T = 0.057\times10^{7}$$

$$\varepsilon_0 = 8.85\times10^{-12}\, farad/m$$

$$\therefore \left(\frac{Z^2}{A}\right)_{critical} = 47.86$$

Let us express the fissionable parameters interms of critical value.

$$\chi = \frac{\left(\frac{Z^2}{A}\right)}{\left(\frac{Z^2}{A}\right)_{critical}}$$

where χ <1 , the nucleus is metastable against the spontaneous fission (stable)

When χ >1 , the nucleus is unstable, against spontaneous fission (unstable)

When χ =1 $\quad \frac{Z^2}{A} = \left(\frac{Z^2}{A}\right)_{critical}$

5.7.Summary: A good model should describe

1. The properties of the ground nuclear states (spins, parities, magnetic dipole, magnetic dipole, electric quadrupole moments etc.)

2.The properties of excited states especially the nuclear excitation spectrum.

3.The dynamical properties of the nucleus.

EXTREME SINGLE PARTICLE MODEL:

1) In this model it is assumed that the nucleons in the nucleus move independently in a common potential, determined by the average motion of all other nucleons.

2). Most of the nucleons are paired so that a pair of nucleons contributes zero spin and zero magnetic moment . The paired nucleus thus form an inert core.

3). The properties of odd A nuclei are characterized by the unpaired nucleon and odd – odd nuclei by the unpaired proton and neutron.

Spin orbit coupling is corresponding to the interaction between the orbital angular momentum and the intrinsic angular moment (spin) of a particle.

Predictions of the shell model:

1)Stability of the closed shell nuclei: This scheme clearly reproduces all the magic numbers.

2)Spins and parities of nuclear ground states:

The semi-empirical mass formula possesses following properties.

1.It explains the stability of the nuclide

2.it explains fission of heavy nuclide

3.it explains some of the regularities associated with alpha decay process

In 1937, N. Bohr proposed the liquid drop model for the nucleus associated with semi empirical mass formula . In this model the finer features of nuclear forces are ignored but the strong inter nucleon attraction is stressed. The essential assumptions are

1) The nucleus consists of incompressible matter so that $R = R_0A^{1/3}$

2) The nuclear force is identical for every nucleon.

3)The nuclear force saturates.

The individual nucleons must be able to move about with in the nucleus much as does an atom of a liquid, therefore think of nucleus as being like a small drop of liquid such a model is known as liquid drop model.

Keywords:
Magic numbers, Shell model, Spin-orbit coupling, Emperical mass formula, Liquid drop model.

Self assessment questions:

1. Explain the need of model for physical system.
2. What are magic numbers? Give some evidences for the existence of magic numbers.
3. Discuss about the nuclear shell model (extreme single particle model). How the shell model predicts the magic numbers.
4. What are the other predictions of shell model?
5. Derive weitz-sacker semi empirical mass formula. How this formula explains the behaviour of nuclear isobars.
6. How does the Bohr-wheeler theory explains the nuclear fission.

Text books

1. Nuclear physics by D.C.Tayal , Himalaya publishing company,Bombay.
2. Nuclear physics by R.C.Sharma, K.Nath&co, Merut
3. Nuclear physics by S.B.Patel

Unit 2

Lesson 6

RADIO ACTIVITY-I

The objectives of the lesson are to explain the following:

6.1. Introduction

6.2 Alpha-Instability

6.3. Theory of Alpha Decay

6.4. Gamma Emission

6.5. Internal Conversion

6.6. Nuclear Isomerism

6.7. Summary.

6.1.Introduction: The phenomenon of radioactive disintegration is the result of instability of nuclei and their tendency to change into more simple species. In terms of binding energy, we can state that a nuclide will be energetically stable towards decay by some specified mode (α-emission, β-emission or spontaneous fission). If its binding energy is greater than the total binding energy of fragments into which it can disintegrate (i.e., if its atomic mass is smaller than the sum of the masses of the products that are formed as a result of decay process.) Such nuclides can of course be disintegrated if energy is supplied to them: the energy supplied appears in the form of kinetic energies of the fragments.

It has been observed that nuclei with $A \geq 140$ are unstable with respect to α- particle emission. This is because the emission of α- particle lowers the coulomb energy -- the principle negative energy contribution to the binding energy of heavy nuclei, but does not change the binding energy appreciably, for α- particle itself is a tightly bound structure.

6.2.Alpha – Instability:

The alpha particle is spontaneously ejected from a nucleus. In the process parent loses an aggregate of two protons and two neutrons.

The process may be expressed as

$$_{Z}X^{A} \longrightarrow {}_{2}He^{4} + {}_{Z-2}Y^{A-4}$$

Where $_{Z}X^{A}$ represents the parent nuclei

$_{Z-2}Y^{A-4}$ represents the daughter nuclei

The process is energetically possible if the sum of the binding energies of last two protons and last two neutrons in the nucleus is less than the alpha binding energy of the value 28.3 Mev.

$$_{Z}X^{A} \longrightarrow {}_{2}He^{4} + {}_{Z-2}Y^{A-4}$$

$$M_p^1C^2 = m_D^1 + T_D + m_\alpha^1 C^2 + T_\alpha \text{ ……………………}(1)$$

M_p^1 is mass of parent nucleus

m_D^1 is mass of daughter nucleus

m_α^1 is mass of α – particle

T_D is kinetic energy of daughter nuclei

T_α is kinetic energy of alpha particle

$$Q = \hat{p}_D + \hat{p}_\alpha \text{.............}(2)$$

Conservation of energy diagram:

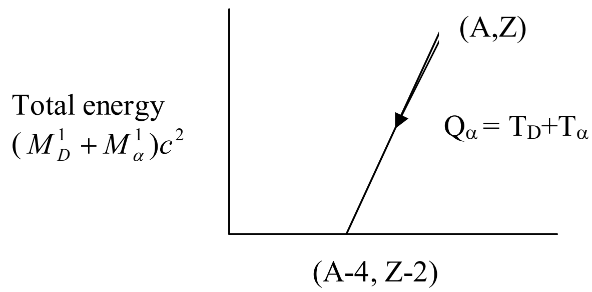


Fig.6.1. Conservation of energy

Conservation of momentum diagram:

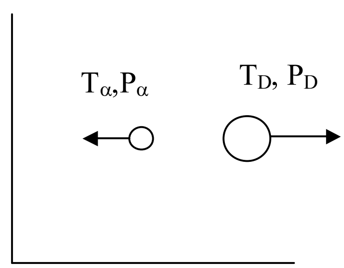


Fig.6.2. Conservation of momentum

$$M_pC^2 = (m_D + m_\alpha) C^2 + T_D + T_\alpha \ldots\ldots\ldots\ldots(3)$$

This is conservation of mass energy eqn in terms of atomic mass energy

$$Q_\alpha = T_D + T_\alpha \ldots\ldots\ldots\ldots(4)$$

$$Q_\alpha = (M_p - (m_D + m_\alpha)) C^2$$

The α particle decay can be measured in two ways

1. Particle spectroscopy 2. mass spectroscopy

$$T_D = \frac{P_D{}^2}{2m_D}, \qquad T_\alpha = \frac{P_\alpha{}^2}{2m_\alpha}$$

$$P_i = P_d + P_\alpha$$

Since in all problems of interest P_i is essentially taken as zero, i.e. it is assumed at rest. Hence $|P_d| = |P_\alpha$

$$T_D{}^2 = \frac{P_\alpha{}^2}{2m_D}$$

$$T_D = \frac{2T_\alpha m_\alpha}{2m_D} = \frac{m_\alpha}{m_D} T_\alpha$$

$$Q_\alpha = T_D + T_\alpha$$

$$= \left(\frac{m_\alpha}{m_D} + 1\right) T_\alpha = \left(\frac{m_D + m_\alpha}{m_D}\right) T_\alpha$$

$$\Rightarrow Q_\alpha = \frac{A}{A-4} T_\alpha \text{..................(5)}$$

$$\Rightarrow T_\alpha = \left(1 - \frac{4}{A}\right) Q_\alpha \text{................(6)}$$

The eqn(6) implies that most of the K.E. liberated in alpha decay is taken away by the α - particle while a small fraction by the daughter nucleus.

$${}_{84}Po^{212} \longrightarrow {}_{2}He^{4} + {}_{82}Pb^{208} + 8.6\text{Mev}$$

$${}_{92}U^{238} \longrightarrow {}_{2}He^{4} + {}_{90}Th^{234} + 4.2\text{Mev}$$

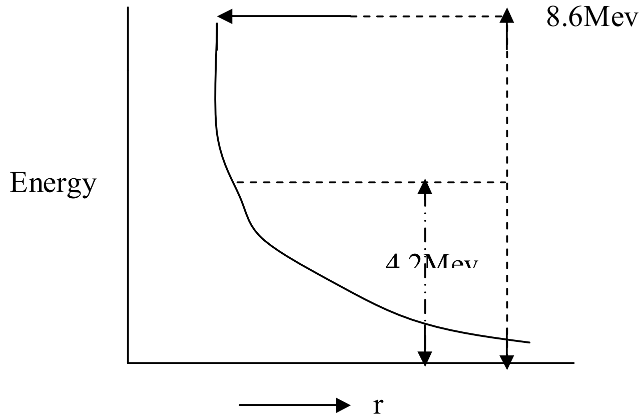


Fig. 6.3.Coulomb energy barrier for Z = 90 and Z = 92

6.3.THEORY OF ALPHA DECAY:

Rutherford in 1927 established that when α - particles of energy of the values 8.8Mev from Po^{213} source are bombarded on a thin U^{238} film, the particles are scattered in accordance with his theory or large angle scattering. Alternately, we can say that they have insufficient kinetic energy to surmount the potential energy barrier arising from the coulomb field round the uranium nucleus. The situation may be illustrated by considering the interaction of α – particle with radioactive nucleus U^{238}, in terms of potential energy curve (fig 1)

As α – particle approaches the nucleus the repulsion between the particle and nucleus increases and at a distance $r = r_1$ it is maximum,

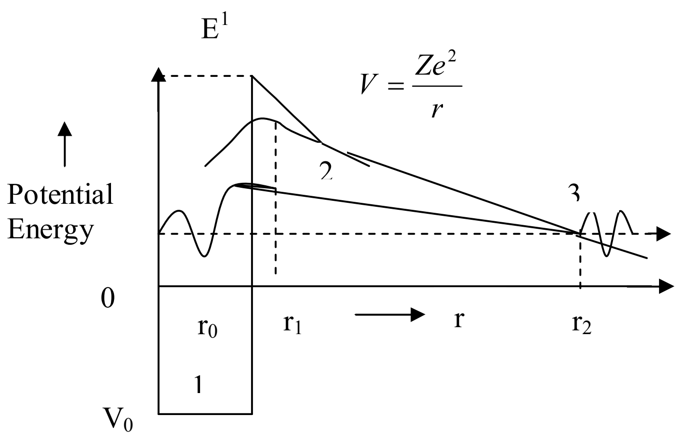


Fig. 6.4.Alpha particle Tunnelling

Near the nucleus and inside the nucleus the shape of the wave is not exactly know, but it is such that in this region we have a constant attraction potential. This is shown by V_0 up to distance r_0 from the nucleus. Classically, the α – particle rests inside the nucleus with kinetic energy equal to $(E + V_0)$ where E is the α – disintegration energy when the particle is far from the nucleus. From fig (1) it turns out that if the particle wants s to escape out of the nucleus it should have maximum energy represented by the curve. Similarly, if an α –particle wants to penetrate the nucleus from outside, again it should have an energy equal to the maximum (E^1) of the curve. Alternately, both way the particle will have to cross an energy barrier of hight E^1. The region between r_2 to r_0 forms the potential barrier because here the potential energy is more than total available energy. As the result of Rutherford experiment we can state the maximum value of potential energy should be greater than 8.8Mev, since α – particle from Po^{214} were scattered by nucleus. However, it is observed that U^{238} spontaneously emits α – particle of energy 4.18Mev.

According to classical ideas, in order to escape from the nucleus of U^{238} .But the energies carried out by alpha particle, emitted by radio-active nuclei, are much lower than the heights of the potential barriers of the respective nuclei. Thus it is very difficult to understand how the particles contained inside the nucleus can go over a potential barrier which is more than twice as high as their total energy. Classical mechanics, provided no explanation of this state. This s can be explain according quantum mechanical theory of barrier penetration. According , to which a particle can leak through a barrier even if its energy is less than the height of barrier. The leakage of the α – particle is termed as α – particle tunneling. The theory of α – particle tunneling was developed by Gamow and Gurney and Condon.

It is assumed that α – particle in the nucleus moves in the spherically symmetric ($l = 0$) field and the wave function is independent of the angle. Then the wave function in the three regions can be written as

$$\frac{d^2\psi}{dr^2} + \frac{2\mu}{\hbar^2}(E - V)\psi = 0$$

with $V = -V_0$ in the region (1)

$$V = \frac{2Ze^2}{r}, \quad with \quad \frac{2Ze^2}{r} \succ E \quad in\ region \quad (2) \quad and$$
$$V = \frac{2Ze^2}{r}, \quad the \quad E \succ \frac{2Ze^2}{r} \quad in \quad region\,(3)$$

In the above expression μ is the reduced mass of the α – particle and is given by $\frac{4}{1+4/A}$

The solution of eqn(1) can be written in form

$$\psi = Ae^{\frac{is}{\hbar}} \ldots\ldots\ldots\ldots\ldots\ldots..(2)$$

where S is variable characteristic of the point to which 'Ψ' belongs and is a function of r. Now

$$\frac{d\psi}{dr} = \frac{Ai}{\hbar} e^{\frac{is}{\hbar}} \frac{\partial s}{\partial r}$$
$$\frac{d^2\psi}{dr^2} = \frac{Ai^2}{\hbar^2} e^{\frac{is}{\hbar}} \left(\frac{\partial s}{\partial r}\right)^2 + \frac{Ai}{\hbar} e^{\frac{is}{\hbar}} \frac{\partial^2 s}{\partial r^2}$$

Putting the values of above derivatives in eqn neglecting the second derivates of S for approximation purpose, we have

$$\frac{Ai^2}{\hbar^2} e^{\frac{is}{\hbar}} \left(\frac{\partial s}{\partial r}\right)^2 + \frac{2\mu}{\hbar^2}(E-V)Ae^{\frac{is}{\hbar}} = 0$$
$$\left(\frac{\partial s}{\partial r}\right)^2 = 2\mu(E-V)$$
$$\frac{\partial s}{\partial r} = \pm\left\{\sqrt{2\mu(E-V)}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(3)$$

The plus sign is retained because we are interested in outgoing wave. Now

$$\int_{r=r_0}^{r=r_2} \frac{\partial s}{\partial r} = \int_{r_0}^{r_2} iPdr \ldots\ldots\ldots\ldots\ldots\ldots..(4) \quad because \qquad P = \sqrt{2\mu(V-E)} \quad since \quad V = \frac{2Ze^2}{r} \succ E$$

$$S_2 - S_0 = \int_{r_0}^{r_2} iPdr\text{.....................}(5)$$

Therefore, the values of the function at $r = r_0$ and $r = r_2$ are given by

$$\psi_0 = e^{\frac{is_0}{\hbar}}$$

$$\psi_2 = e^{\frac{is_2}{\hbar}}\text{....................}(6)$$

Decay probability:

The probability of finding the particle at the extreme of the potential barrier will be given by

$$P = \frac{|\psi_2|^2}{|\psi_0|^2}$$

$$= e^{2i/\hbar(S_2 - S_0)}$$

$$= e^{2i/\hbar\int_{r_0}^{r_2} iPdr}$$

$$= e^{-2\int_{r_0}^{r_2}\frac{P}{\hbar}dr}$$

$$= e^{-2\int_{r_0}^{r_2}\frac{1}{\hbar}\sqrt{2\mu\left(\frac{2Ze^2}{r}-E\right)}dr}\text{..................}(7)$$

$$= e^{-2G}\text{......................}(8)$$

$$where \qquad G = \sqrt{\frac{2\mu}{\hbar^2}\left(\frac{2Ze^2}{r} - E\right)}\text{..........................}(9)$$

The eqn (8) gives the probability of α – particle tunneling

Geiger Nuttal Rule: At $r = r_2$ the potential energy and kinetic energy of α – particle are equal, hence $\frac{2Ze^2}{r_2} = E$ therefore

$$G = \int_{r_0}^{r_2} \sqrt{\frac{2\mu}{\hbar^2}\left(\frac{2Ze^2}{r} - \frac{2Ze^2}{r_2}\right)}dr$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(10)$$

$$G = \int_{r_0}^{r_2} \sqrt{\frac{2\mu E}{\hbar^2}\left(\frac{r_2}{r} - 1\right)}dr$$

The dependence of 'G' over 'E' implies that probability of α – emission depends upon the disintegration energy. The eqn (10) can be written as

$$G = \int_{r_0}^{r_2} \sqrt{\frac{4Ze^2\mu}{\hbar^2}\left(\frac{1}{r} - \frac{1}{r_2}\right)}dr \ldots\ldots\ldots\ldots\ldots\ldots\ldots.(10(a))$$

Taking only $g = \int_{r_0}^{r_2} \left(\frac{1}{r} - \frac{1}{r_2}\right)^{1/2} dr$

Put

$$r = r_2 Cos^2\theta \Rightarrow \theta = Cos^{-1}\sqrt{\frac{r}{r_2}}$$

$$dr = -2r_2 Cos\theta \sin\theta d\theta$$

$$\int \left(\frac{1}{r} - \frac{1}{r_2}\right)^{1/2} dr = \int \left(\frac{1}{r_2 Cos^2\theta} - \frac{1}{r_2}\right)^{1/2} \left(-2r_2 Cos\theta Sin\theta d\theta\right)$$

$$= -2\sqrt{r_2}\int \frac{\left(1 - Cos^2\theta\right)^{1/2}}{Cos\theta} \cos\theta \; Sin\theta \;\; d\theta$$

$$= -2\sqrt{r_2}\int \sin^2\theta \; d\theta$$
$$= -2\sqrt{r_2}\int \frac{1-Cos2\theta}{2} d\theta$$
$$= -2\sqrt{r_2}\left[\frac{1}{2}\left(\theta - \frac{1}{2}Sin2\theta\right)\right]_{\theta=Cos^{-1}\sqrt{\frac{r_0}{r_2}}}^{0}$$

Hence

$$\int_{r_0}^{r_2}\left(\frac{1}{r}-\frac{1}{r_2}\right)^{1/2} dr = -\sqrt{r_2}\left[Cos^{-1}\sqrt{\frac{r}{r_2}} - \sqrt{\left(1-\frac{r}{r_2}\right)}\sqrt{\frac{r}{r_2}}\right]_{r_0}^{r_2}$$
$$= -\sqrt{r_2}\left[Cos^{-1}1 - Cos^{-1}\sqrt{\frac{r_0}{r_2}} + \sqrt{\left(1-\frac{r_0}{r_2}\right)}\sqrt{\frac{r_0}{r_2}}\right]$$

$$Taking \qquad Cos^{-1}1 = 0$$

$$g = -\sqrt{r_2}\left[-\left(Cos^{-1}\sqrt{\frac{r_0}{r_2}} - \sqrt{\left(1-\frac{r_0}{r_2}\right)}\sqrt{\frac{r_0}{r_2}}\right)\right]$$
$$= \sqrt{r_2}\left[Cos^{-1}\sqrt{\frac{r_0}{r_2}} - \sqrt{\left(1-\frac{r_0}{r_2}\right)}\sqrt{\frac{r_0}{r_2}}\right] \qquad \text{.....................................(11)}$$

Substituting this value in eqn (10(a))

$$G = \sqrt{\frac{4\mu Ze^2 r_2}{\hbar^2}}\left[Cos^{-1}\sqrt{\frac{r_0}{r_2}} - \sqrt{\frac{r_0}{r_2} - \frac{r_o^{\;2}}{r_2^{\;2}}}\right]$$
$$Let\sqrt{\frac{r_0}{r_2}} = x$$

$$G = \sqrt{\frac{4\mu Ze^2 r_2}{\hbar^2}}\left(Cos^{-1}x - x\sqrt{1-x^2}\right)$$

$$For \quad small \quad values \quad of \ x \qquad Cos^{-1}x = \frac{\pi}{2} - x$$

$$G = \sqrt{\frac{4\mu Ze^2 r_2}{\hbar^2}}\left(\left(\frac{\pi}{2} - x\right) - x\right)$$

$$G = \sqrt{\frac{4\mu Ze^2 r_2}{\hbar^2}}\frac{\pi}{2}\left(1 - \frac{4x}{\pi}\right)$$

The Kinetic energy of the α- particle is given by

$$E = \frac{1}{2}\mu_\alpha v_\alpha{}^2 = \frac{2Ze^2}{r_2}$$

$$r_2 = \frac{4Ze^2}{\mu_\alpha v_\alpha{}^2}; \qquad v_\alpha = Velocity \ of \quad the \ \ \alpha \quad - particle$$

$$G = \sqrt{\frac{4\mu ze^2}{\hbar^2}\left(\frac{4Ze^2}{\mu V_\alpha^2}\right)}\frac{\pi}{2}\left(1 - \frac{4}{\pi}\sqrt{\frac{r_0\mu_\alpha V_\alpha^2}{2Ze^2}}\right)$$

$$G = \frac{4Ze^2\pi^2}{h \ v_\alpha}\left[1 - \frac{2}{\pi}\sqrt{\left(\frac{\mu_\alpha v_\alpha{}^2}{Ze^2}\right)}\right]$$

$$G = \frac{4Ze^2\pi^2}{h \ v_\alpha} - \frac{8\pi e}{h}\sqrt{\mu Zr_0} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(12)$$

$$\therefore P = e^{-2\left[\frac{4\pi^2 Ze^2}{hv_\alpha} - \frac{8\pi e}{h}\sqrt{\mu Zr_0}\right]} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(13)$$

Frequency of striking the barrier:

Now ‘n’ the frequency of α – particle with which it presents itself as the barrier is written as

$$n = \frac{v_\alpha}{2r_0} \qquad \text{...................................(14)}$$

The theorictical value of this expression comes out to be $\frac{\hbar}{4m_\alpha r_0{}^2}$ and it is numerically equal to 10^{21} collisions counts. This makes the decay constant λ to be written in the form

$$\lambda = nP$$

$$= e^{-2\left[\frac{4\pi^2 Ze^2}{hv_\alpha} - \frac{8\pi e}{h}\sqrt{\mu Zr_0}\right]} \qquad \text{........................(15)}$$

$$\log_e^\lambda = \log_e^{\frac{v_\alpha}{2r_0}} - 2\left[\frac{4\pi Ze^2}{hv_\alpha} - \frac{8\pi e}{h}\sqrt{\mu Zr_0} \text{................(16)}\right]$$

Now if a graph is plotted between the logarithm of decay constant and reciprocal of α – particle velocity, the graph is straight line and eqn is regarded as the theoretical form of Geiger Nuttal law It is to be noted that when α – particle is emitted, the interaction is between daughter nucleus and α – particle and therefore in the above expression Z should be replaced by (Z –2) and we write

$$\log_e^\lambda = \log_e^{\frac{v_\alpha}{2r_0}} - 2\left[\frac{4\pi(Z-2)e^2}{hv_\alpha} - \frac{8\pi e}{h}\sqrt{\mu(Z-2)r_0}\right] \text{....................(17)}$$

The expression for the average period is

$$\tau = \frac{1}{\lambda} = \frac{2r_0}{v_\alpha}\exp\left[2\left\{\frac{4\pi(Z-2)e^2}{hv_0} - \frac{8\pi e}{h}\sqrt{\mu(Z-2)r_0}\right\}\right] \text{........................(18)}$$

Some Remarks About Alpha Decay:

1)If 'Q' the energy of the α – particle and λ are known for the given nucleus, and the values of r_0 is calculated then it is found that the results are in good agreement with the formula $r = r_0 A^{1/3}$

In addition to this graph between - and reciprocal of $V_{\alpha}j$ is straight line. These two results confirm experimentally, Gamow theory of α – decay.

6.4.Gamma Emission:

The term gamma – rays is used to include all electromagnetic radiations emitted by radioactive substance. The spectral region which gamma rays occupy ranges from soft X – ray region to very short wavelength of the order of few x units.$(1x^0 = 10^{-11}cm)$

Properties of Gamma Rays:

1. The energy of gamma – rays is characterized by ‘hv’ , which may be expressed in ergs, ‘ev’ or in the unit of m_0c^2
2. The gamma – rays do not bend in electric and magnetic fields.
3. They travel with the velocity of light and made to diffract and interfere just as X –rays.
4. The gamma – rays are having very high penetrating power.
5. They produce less ionization per unit length of the path.

To find the energy of gamma – rays , generally we can use the following three methods They are

a) Demond crystal spectrometer

b) Measurement of moderate energies.

c) Pair spectrometer.

Multipole Radiations: Gamma radiations can be divided in to two general categories, namely electric and magnetic radiations. Electric radiations a rise from changes in the distribution of the electric changes in the nucleus, where as magnetic radiation arise from change sin the distribution of the magnetic poles or in the current distribution in the nuclei

For complete description of this radiation, requires the quantum theory of radiation. In this, we shall make the assumptios, as follows,

a)The wavelength of the emitted radiation is much larger than the nuclear radius.

b)The radiating system consists of individual nucleons which contribute to the radiation due to the motion of their charges and magnetic moments.

Let us consider the electromagnetic field in a uniform isotropic loss less medium i.e. the free space region free of sources of radiation. The maxwell's equation for E and B vectors are,

$$\begin{aligned} &\nabla . E = 0 \\ &\nabla . B = 0 \\ &\nabla \times E = -\frac{\partial B}{\partial t} \qquad \text{..................(1)} \\ &\nabla \times B = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} \end{aligned}$$

There electric and magnetic fields are continuous vector functions of position and time. Thus time dependence is sinusoidal and fields can be written as

$$\left.\begin{aligned} E(r,t) &= E(r)\, e^{-i\omega t} + E^*(r)\, e^{i\omega t} \\ B(r,t) &= B(r)\, e^{-i\omega t} + B^*(r)\, e^{i\omega t} \end{aligned}\right. \text{......................(2)}$$

On substitution of eqn(2) in eqn (1), we get,

$$\nabla \times E = -\frac{\partial B}{\partial t} = i\omega B \text{......................(3)}$$

$$and \qquad \nabla \times B = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} = -i\omega \mu_0 \varepsilon_0 E \text{........................(4)}$$

From (3) &(4),

$$B = \frac{1}{i\omega}(\nabla \times E)$$
$$E = \frac{1}{-i\omega\mu_0\varepsilon_0}(\nabla \times B) \qquad \ldots\ldots\ldots\ldots\ldots\ldots(5)$$

Using eqn(5), we can write eqn (3) & (4) as follows.

$$\nabla \times \left(\frac{1}{-i\omega\mu_0\varepsilon_0} \nabla \times B \right) = i\omega B$$
$$\nabla \times \nabla \times B = \omega^2 \mu_0 \varepsilon_0 B \quad \ldots\ldots\ldots\ldots\ldots\ldots(6)$$

and

$$\nabla \times \left(\frac{1}{i\omega} \nabla \times E \right) = -i\omega\mu_0\varepsilon_0 E$$
$$\nabla \times \nabla \times E = \omega^2 \mu_0 \varepsilon_0 E \quad \ldots\ldots\ldots\ldots\ldots\ldots(7)$$

From (6) & (7),

$$(\nabla \times \nabla \times -k^2)B = 0$$
$$(\nabla \times \nabla \times -k^2)E = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots(8)$$

where $k^2 = \omega^2 \mu_0 \varepsilon_0$ *and* $(\nabla \times \nabla \times -k^2)$ *is an operator*

Therefore the vector eigen functions of the operator $(\nabla \times \nabla \times -k^2)$ have the form

$cLj(kr)\, Y(\theta, \phi)$ where c is a constant.

If we thus set $B_e(L, M) = c^{(e)} Lj(kr)\, Y(\theta, \phi)$......................(9)

Then 'B' has now radial component , such fields are called electric multipole fields, hence the index (e) is used.

Therefore from eqn(5), the electric field strength given by

$$E_e(L,M) = -\frac{1}{i\omega\mu_0\varepsilon_0} c^{(e)} \nabla \times Lj(kr)Y(\theta,\phi) \text{.......................}(10)$$

If on the other hand from eqn(8) , we get,

$$E_m(L,M) = c^{(m)} Lj(kr)Y(\theta,\phi) \text{.........................}(11)$$

Then 'E' has no radial component such fields are called magnetic multipole fields. The corresponding magnetic field by using eqns(5) &(11).is

$$B_m(L,M) = \frac{1}{i\omega} c^{(m)} \nabla \times Lj(kr)Y(\theta,\phi) \text{......................}(12)$$

In the above eqns, the values of constants $c^{(e)}, c^{(m)}$ can be determined by using the normalization of fields. Since an electromagnetic wave contains on the average half of its energy in the magnetic field and half in the electric field, hence,

$$\varepsilon_0 \int |E|^2 dv = \frac{1}{\mu_0} \int |B|^2 dv = \frac{1}{2}\hbar\omega = \frac{1}{2}\hbar ck \text{.....................}(13)$$

For electric multipole radiation, eqn(9) can be inserted in eqn(13).

$$\frac{1}{\mu_0} \int_0^{R_0} \left[c^{(e)}\right]^2 |j(kr)|^2 r^2 dr \int (LY(\theta,\phi))^* (LY(\theta,\phi)) d\Omega = \frac{1}{2}\hbar ck \text{...........}(14)$$

Where R_0 is the radius of sphere (nucleus).

Since 'L' is a Hamiltonian operator, we obtain that the angular part of the integral is

$$\int (LY(\theta,\phi))^* (LY(\theta,\phi)) d\Omega = \int Y^* L^2 Y d\Omega = L(L+1)\hbar^2 \text{..................}(15)$$

For

$$r \to \alpha, \qquad j(kr) = \frac{1}{kr}\cos\left[kr - \frac{1}{2}\pi(L+1)\right]$$

$$\therefore \int_0^{R_0} |j(kr)|^2 r^2 dr = \frac{1}{k^2}\int_0^{R_0} \cos^2(kr - \frac{1}{2}\pi(L+1)dr$$

$$\approx \frac{R_0}{2k^2} \quad \text{.............................(16)}$$

substitute eqns (15),(16) in eqn (14), we get,

$$\frac{1}{\mu_0}\left[c^{(e)}\right]^2 \frac{R_0}{2k^2} L(L+1)\hbar^2 = \frac{1}{2}\hbar ck$$

$$\left[c^{(e)}\right]^2 = \frac{1}{2}\hbar ck \times \frac{\mu_0 2k^2}{R_0 L(L+1)\hbar^2}$$

$$\left[c^{(e)}\right]^2 = \frac{ck^3\mu_0}{\hbar R_0 L(L+1)} \quad \text{.....................(17)}$$

Similarly, we obtain for magnetic multipole radiation , by using eqn (11),(13),we get

$$\left[c^{(m)}\right]^2 = \frac{ck^3\varepsilon_0}{\hbar R_0 L(L+1)} \quad \text{...................(18)}$$

These two constants values are substituting in equns (9),(10),(11) & (12) these are the multipole expansions for electric and magnetic field are

$$B_e(L,M) = \left[\frac{ck^3\mu_0}{\hbar R_0 L(L+1)\hbar}\right]^{1/2} Lj(kr)Y(\theta,\phi)$$
$$E_e(L,M) = -\frac{1}{i\omega\mu_0\varepsilon_0}\left[\frac{ck^3\mu_0}{\hbar R_0 L(L+1)\hbar}\right]^{1/2} \nabla\times Lj(kr)Y(\theta,\phi)$$
$$E_m(L,M) = \left[\frac{ck^3\mu_0}{\hbar R_0 L(L+1)}\right]^{1/2} Lj(kr)Y(\theta,\phi)$$
$$B_m(L,M) = \frac{1}{i\omega}\left[\frac{ck^3\mu_0}{\hbar R_0 L(L+1)\hbar}\right]^{1/2} \nabla\times Lj(kr)Y(\theta,\phi)$$

For L = 0 , the mltlipole fields are vanish identically, this is a consequence of the transverse nature of a light wave in free space.

Therefore, any arbitary field E(r) and B(r) can be expanded in terms of multipoles

$$E(r) = \sum_{l=1}^{\infty}\sum_{m=-l}^{l}\left[a_e(L,M)E_e(L,M;r) + a_m(L,M)E_m(L,M;r)\right]$$
$$B(r) = \sum_{l=1}^{\infty}\sum_{m=-l}^{l}\left[a_e(L,M)B_e(L,M;r) + a_m(L,M)B_m(L,M;r)\right]$$

where the coefficients $a_e(L,M)$ *and* $a_m(L,M)$ are the amplitude of the electric and the magnetic $2^l -$ poles respectively.

SELECTION RULES: The selection rules for emission of electric or magnetic multipole radiation may be obtained from the angular momentum and parity of the field.

1. The conservation of energy implies that the difference of energy in initial and final should be given by ħώ
2. The conservation of charge requires that initial and final states should have the same charge, since none is carried off by photons.
3. The conservation of angular momentum requires the difference of angular momentum I_i of initial and I_f of final state should be equal to $l\hbar$. The difference between two

momenta ranges from $| I_i - I_f |$ to $| I_i + I_f |$, Hence the selection rule for angular momentum for both electric and magnetic radiation can be written as $| I_i + I_f | \geq l \leq | I_i - I_f |$. For example if transition is between $I_i(4^+) to\ I_f(2^+)$ then l can have values $| 4 - 2 |$ to $| 4 + 2 |$ i.e. l values from 2 to 6.

4. Now if initial and final states have the same parity electric multipoles of even l and magnetic multipoles of odd l are allowed.

If initial and final states have opposite parities, electric multipoles of odd l and magnetic multipoles of even are allowed.

6.5.INTERNAL CONVERSION: The internal conversion results due to electromagnetic interaction between excited nucleus and an orbital electron. In internal conversion the energy of interaction ejects one of the orbital electron.

The excited nucleus surrenders its energy to the orbital electron and hence the kinetic energy of the conversion electron, T_e will given by

$$T_e = (E_i - E_f) - I_i \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Where E_i and E_f are the energies of initial and final levels and I_i is the binding energy of electron in its orbit.

The internal conversion electrons produce a series of mono energetic lines and not a continuous spectrum as in β^- emission

The line with lowest energy is

$$T_{ek} = (E_i - E_f) - I_k \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

The next line is $T_{el} = (E_i - E_f) - I_l \ldots\ldots\ldots\ldots\ldots\ldots(3)$

And so on.

Conversion coefficient: The two processes γ – emission and internal conversion often complete with one another. The relationship between the two de excitation processes is expressed by the internal conversion coefficient α . which is equal to ratio of the probability of conversion electron (P_e) to the probability of γ – emission (P_g) i.e.

$$\alpha = \frac{P_e}{P_g} \qquad \text{..................(4)}$$

$$or \quad \alpha = \frac{\lambda_e}{\lambda_g} \qquad \text{.............(5)}$$

eqns (4) & (5) also measures the total no of conversion electrons emitted over a given time divided by the total no. of gamma photons emitted in the same transition in the same time.

The conversion coefficient α can be expressed as

$$\alpha = \alpha_k + \alpha_L + \alpha_M + \ldots\ldots\ldots\ldots \qquad \ldots\ldots\ldots\ldots\ldots\ldots(6)$$

Where α_k , α_L, α_M are partial conversion coefficients of k,L, ….electrons, respectively

The measurement of conversion coefficient provides information about ($E_i - E_f$), for the energy of conversion electrons is also expressed as

$$T_e = E_g = (E_i - E_f) - I_i \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

6.6.NUCLEAR ISOMERISM: Theoritical estimates indicate that normally the life time of γ – transition is of the order of 10^{-13} second. But there are about 250 known cases, in which life-time ranges from 10^{-10} sec to several years. In such cases nucleus remain is the excited state for measurable time and then decay. These delayed transitions are called isomeric transitions and the states from which they originate are called isomeric states or isomeric levels. Nuclear species which have the same atomic and mass numbers, but have different radioactive properties, are called nuclear isomers and their existence is referred to as nuclear isomerism.

The first example of nuclear isomerism and is shown in fig.

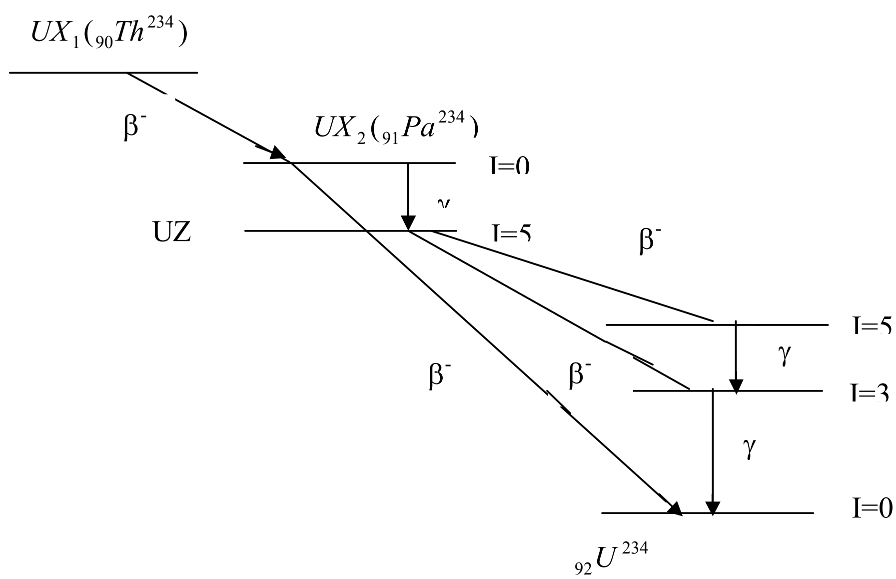


Fig. 6.5.Nuclear Isomerism of UX_2 and UZ

The beta emission of $UX_1(_{90}{}^{Th234})$ gives an isomeric pair UX_2 and UZ of $(_{91}Pa^{234})$. The transition of UX_2 to UZ by gamma emission is highly for bidden, since $\Delta I = 5$. Therefore, the usual decay process for UX_2 is beta decay giving rise to $_{92}U^{234}$.How ever a small percentage (0.12%) of UX_2 goes to UZ by gamma emission. UZ nucleus transforms to $_{92}U^{234}$ beta decay which leaves the nucleus in excited states. The de-excitation takes place by the emission of γ-rays.

Classification of Nuclear Isomer: Nuclear Isomers may be classified as

a) isomer with independent decay

b) Genetically related Isomers

c) Isomers of stable nuclei.

a)Isomers with independent decay: In this type , each isomer decays independently of the other with its own particular half life. The transition from the metastable to the ground state is highly forbidden. In figure T_1 and T_2 are the transition with the different half lives.

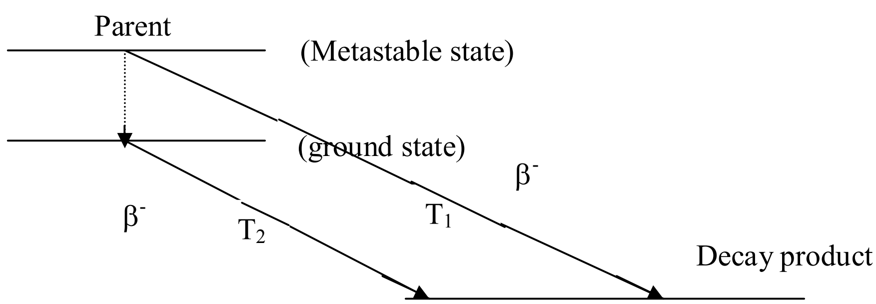


Fig. 6.6.Nuclear isomerism with independent Decay

b)Genetically related Isomers: In this type, the metastable state decays to the ground state with a definite half-life T_i

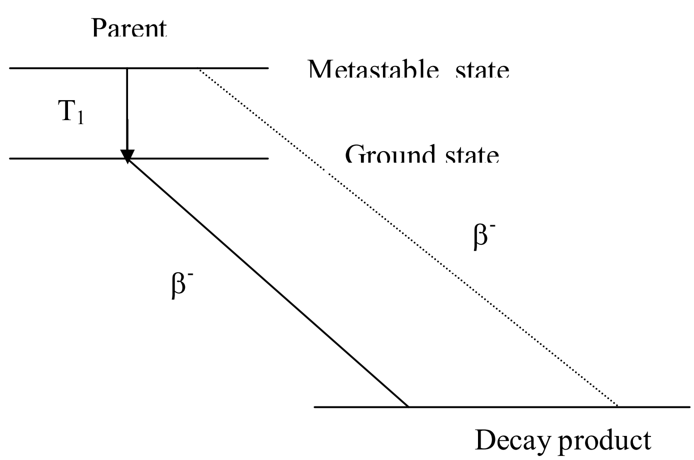


Fig. 6.7.Genetically related Isomers

Mostly, the gamma radiation is internally converted and produces line spectrum of electrons together with characteristic X- rays. The ground state decays to form the product with a half-life of T_2 different from T_1

c)Isomers of stable nuclei: In this type of isomers, the decay process involves an isomeric transitions from the metastable state to the ground state of a stable nuclide , accompanied by the emission of gamma radiation. More than 30 stable species are found in nature.

6.7.Summary:

It has been observed that nuclei with A$\geq$ 140 are unstable with respect to α- particle emission. This is because the emission of α- particle lowers the coulomb energy -- the principle negative energy contribution to the binding energy of heavy nuclei, but does not change the binding energy appreciably, for α- particle itself is a tightly bound structure.

Rutherford in 1927 established that when α - particles of energy of the values 8.8Mev from Po^{213} source are bombarded on a thin U^{238} film, the particles are scattered in accordance with his theory or large angle scattering. Alternately, we can say that they have insufficient kinetic energy to surmount the potential energy barrier arising from the coulomb field round the uranium nucleus. The term gamma – rays is used to include all electromagnetic radiations emitted by radioactive substance. The spectral region which gamma rays occupy ranges from soft X – ray region to very short wavelength of the order of few x units.($1x^0 = 10^{-11} cm$)

Gamma radiations can be divided in to two general categories, namely electric and magnetic radiations. Electric radiations a rise from changes in the distribution of the electric changes in the nucleus, where as magnetic radiation arise from change sin the distribution of the magnetic poles or in the current distribution in the nuclei

The internal conversion results due to electromagnetic interaction between excited nucleus and an orbital electron. In internal conversion the energy of interaction ejects one of the orbital electron.

Theoritical estimates indicate that normally the life time of γ – transition is of the order of 10^{-13} second. But there are about 250 known cases, in which life-time ranges from 10^{-10} sec to several years. In such cases nucleus remain is the excited state for measurable time and then decay. These delayed transitions are called isomeric transitions and the states from which they originate are called isomeric states or isomeric levels.

Keywords:
Spontaneous fission, barrier penetration, alpha decay, nuclear isomerism

Self assessment questions:

1. Give an account of alpha decay on the basis of quantum mechanical theory of barrier penetration.
2. What are the properties of gamma rays?
3. Explain in detail about the "nuclear isomerism" and "nuclear conversion"..

Text books

4. Nuclear physics by D.C.Tayal , Himalaya publishing company,Bombay.
5. Nuclear physics by R.C.Sharma, K.Nath&co, Merut
6. Nuclear physics by S.B.Patel

Unit 2

Lesson 7

RADIO ACTIVITY II

The objectives of the lesson are to explain the following:

7.1 Introduction

7.2. Beta Decay
7.3. The Neutrino Hypothesis
7.4. Fermi Theory of Beta Decay
7.5. Selection Rules
7.6. Summary

7.1.Introduction: The portion of the radiation, emitted from a radioactive source, that was strongly deflected by perpendicular magnetic field was termed beta-radiation. There are three modes of beta radiation : negatron (β^-) emission, orbital electron capture and positron (β^+) emission. Negatron emission is much more common than t he other decay processes. Beta rays are easily distinguished from α- particles by their considerably greater range in matter. When a radio-element emits a β^- particle , the product has the same mass number as the parent, but its atomic number increases by one unit. Similarly when a positron is emitted the mass number is still unchanged but the atomic number of the product is now one unit less than that of the parent. When the ratio of neutrons to protons is low , another type of decay known as orbital electron capture process has been found to occur. In this process instead of a proton being converted into a neutron with an emission of a positron, electrons are captured by the nucleus from the first (usually) or any other quantum level, which combines immediately with a proton to form a neutron. The product of this type would have the same mass number as its parent but its atomic number would be one unit lower as in the case of positron emission.

7.2.BETA DECAY: The spontaneous decay process in which mass number of nucleus remain unchanged, but atomic number changes, is termed as β – decay. The change in atomic number is accomplished by the emission of an electron, emission of a position or by the capture of an orbital electron. Thus , depending upon the three modes of decay, the β – decay are known as β^- decay, β^+ decay and electron capture (k- capture). The half lives of β – active nuclei range between 0.06sec to 10^{18} years. The energy of emitted particle goes up to few Mev.

THREE FORMS OF BETA DECAY: when a nucleus has an excess of neutrons it is, in general, unstable with respect to nucleus which has got the same mass number but a greater number of proton. This occurs by emission of nuclear origin is termed as β^- particle. The β^- decay is energetically possible according to the binding energy concept when

$${}_ZM^{1A} > {}_{Z+1}M^{1A} + m_e \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1a)$$

$$\text{or } {}_ZM^{A} > {}_{Z+1}M^{A} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2a)$$

Where ${}_ZM^{1A}$ and ${}_{Z+1}M^{1A}$ are the masses of parent and daughter nuclei,

m_e is mass of electron and ${}_ZM^{A}$ and ${}_{Z+1}M^{A}$ are the masses of parent and daughter atoms.

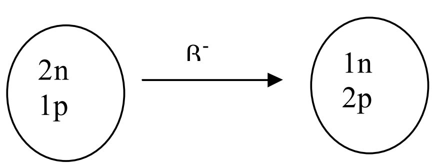
simple example is

$${}_1H^3 \xrightarrow{\beta^-} {}_2He^3$$

$$n \longrightarrow p + e^- \ldots\ldots\ldots\ldots\ldots\ldots(3a)$$

$$P \longrightarrow n + e^+ \ldots\ldots\ldots\ldots\ldots\ldots(4a)$$

There fore, we can say that in β^- decay a neutron is changed in to proton with subsequent emission of an electron. The example of eqn(3) is shown in figure.



Positron Emission: The atomic number is decreased either by the emission of positive electron by the capture of an orbital electron . The positron emission takes place when an excess proton is changed in to neutron according to

$$P \longrightarrow n + e^{+} \ldots\ldots\ldots\ldots\ldots (1b)$$

The process is energetically possible when

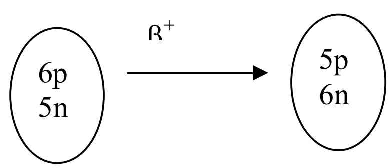
$$_{Z+1}M^{1A} > {}_{Z}M^{1A} + m_e \ldots\ldots\ldots\ldots (2b)$$

$$_{Z+1}M^{A} > {}_{Z}M^{A} + 2\, m_e \ldots\ldots\ldots\ldots (3b)$$

The β^+ decay was discovered later than β^- decay because all naturally occurring nuclides have an excess of neutrons.

For example $_6C^{11} \xrightarrow{\beta^+} {}_5B^{11} \ldots\ldots\ldots\ldots (4b)$

The above example is shown in the following figure.



Electron capture:-

When unstable nuclides are proton rich, the coulomb barrier tends to prevent the emission of a positron. The only alternative left for an unstable nucleus to transform in to a stable nucleus by the capture of an orbital electrons. There by, transforming proton in to neutron and decreasing the atomic number. This process is known as electron capture. This mode of decay was discovered in 1938 by Alvarez. The electron capture is important in heavy nuclei in which K subshell lies near to the nucleus and there fore K capture is more probable than 'L' capture and M capture.

The K-capture is energetically possible when

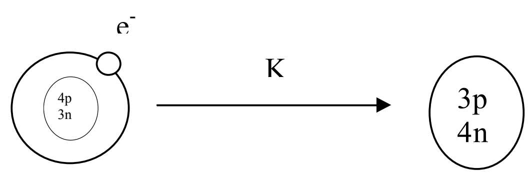
${}_{Z}M^{1A} < {}_{Z+1}M^{1A} + m_e$ ……………..(1c)

${}_{Z}M^{A} < {}_{Z+1}M^{A}$…………….(2c)

The simple example of K-capture is

$${}_{4}Be^{7} \xrightarrow{k} {}_{3}Li^{7} \text{…………(3c)}$$

and is illustrated in figure.

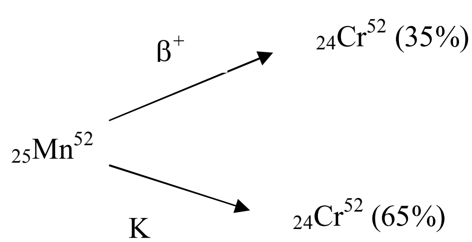


Some remarks about three forms of decay:

The close examination of the inequalities (2a, 3b, 3c) reveals the following points.

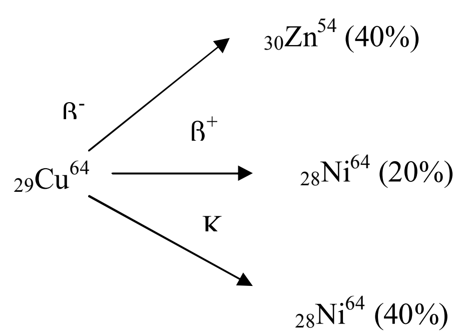
if ${}_{Z}M^{A} > {}_{Z+1}M^{A}$; the nucleus is β^- active and for ${}_{Z}M^{A} < {}_{Z+1}M^{A}$ the nucleus is β^+ active. It means that no stable isobars with charges differing by unity are possible.

When inequality (3b) is satisfied, the condition (2c) is automatically fulfilled . It implies t hat two processes (β^+ decay and K – capture) may run simultaneously. The example is



It is observed that ratio of running two process depends on atomic number 'Z'. The ratio increases with increasing 'Z' for given decay energy. There fore, β^+ emission in heaviest element is very rare.

The isobars $_{z+1}R^A$ and $_{z+1}S^A$ may satisfy the inequalities (2a, 3b) for some given nuclei, then all the three forms of disintegrations will be observed. As for example



7.3.THE NEUTRINO HYPOTHESIS: The continuous energy distribution of electrons in β - decay proved to be a great puzzle, although the maximum energy of the distribution corresponds to that expected from the mass difference of the parent and the daughter. There is also an apparent failure to conserve linear and angular momentum in β - decay. The emitted electron does not travel in a direction opposite to that of product nucleus. The angular momentum and statistics are not conserved.

All these difficulties were eliminated by Pauli in 1933 by assuming the existence of an additional particle. This hypothetical particle is called neutrino. To preserve not only the principle of energy conservation but also the principle of conservation of electric charge and angular momentum and the rules governing statistics we must ascribe Certain properties to the neutrino. Charge is already conserved by the disintergarion electron in β - decay hence the neutrino has zero charge. In β - decay the parent and daughter nuclei always have the same mass number. This require that both have the same mass number. This require that both have the same statistics and that their nuclear angular momenta may differ only by zero or an integer multiple of h/2π. As the beta ray electron has Fermi-dirac statistics and spin ħ/2, the neutrino must also have Fermi –Dirac statistics and an intrinsic spin ħ/2 in order to conserve statistics and angular momentum. In addition to a neutrino. An antiparticle called anti-neutrino. Should also exist. The neutrino and antineutrino are represented as υ and $\ddot{\upsilon}$. The antineutrino has the same mass, spin, charge and magnitude of magnetic moment as those of neutrino. In all processes where a neutrino is emitted an antineutrino can be absorbed with the same result and vice versa. IN the neutrino the spin and angular momentum vector are oppositely directed and in the antineutrino these vectors are aligned together.

Neutrino is emitted with β^+ emission. Antineutrino is emitted with β^- emission. Neutrino is emitted with orbital electron capture.

$$n \longrightarrow P + e^- + \ddot{\upsilon}$$

$$P \longrightarrow n + e^+ + \upsilon$$

$$P + e^+ \longrightarrow n + \upsilon$$

The electron , neutrino and product nucleus share among them the energy, momentum and angular momentum available from the nuclear transitions. The β – particle gets maximum energy when the neutrino is emitted with zero momentum.

7.4.FERMI THEORY OF BETA DECAY: In 1934, Fermi made a successful theory of beta decay based on Pauli's neutrino hypothesis. This theory is based on the following assumptions.

a) The light particles the electron and neutrino are created by the transformation of neutron into a proton in a nucleus or vice versa.

b) The energy remains conserved in the decay process, the available energy being shared among the electron and the neutrino. Due to larger mass product nucleus does not receive K.E.

c) The neutrino has rest mass zero, or very small compared to that of the electron.

d) The β – decay process is analogous to the emission of electromagnetic radiation by an atom with the electron neutrino field acting in place of electromagnetic field.

e) Electron – neutrino field is weak in contrast to the short range strong interaction which exist between the nucleons bound in the nucleus.

f) Time dependent perturbation theory is very good approximation because of the smallness of coupling constants.

g) No nuclear parity change occurs

h) Ass nucleons move with velocities of only ~ c/10 in nuclei. Calculations can be made with non-relativistic nuclear wave functions.

Using Dirac's expression for the transition probability per unit time of an atomic system to emit photon, using time dependent perturbation theory, the probability that an electron of momentum between P_e and $P_e + dP_e$ is emitted per unit time may be written as

$$P(P_e)dP_e = \frac{2\pi}{\hbar}\left|H_{if}\right|^2 \frac{dN}{dE_0} \text{..................}(1)$$

where $\frac{dN}{dE_0}$ is the number of quantum mechanical states of final system per unit energy interval. H_{if} the matrix element of the interaction for the initial & final states.

Interaction matrix element. It is defined as

$$H_{if} = \int \psi_f^* H \psi_i d\tau \text{...................}(2)$$

Where Ψ_f and Ψ_i respectively are the wavefunctions of the system in its final state and in its initial state. H is Hamiltonian operator that describes the weak interaction between the two states and $d\tau$ is the volume element

For the negatron decay n $\longrightarrow$ P + $_{-1}e^0$ + ῡ

We have Ψ_i = Ψ (parent nucleus) = Ψ_i

Ψ_f = Ψ (daugther nucleus) Ψ (electrton) Ψ (antineutrino)

= $\Psi_f \Psi_e \Psi_{\bar{\upsilon}}$

we do not know the form of interaction operator H, but Fermi suggested a new constant called as Fermi coupling constant denoted by g.

The emission of neutrino and the absorption of an anti-neutrino of opposite momentum are equivalent we may replace $\psi_{\bar{\nu}}{}^{*}$ *and* ψ_ν to make equation more symmetrical

$$H_{if} = g\int\left[\psi_f^*\psi_e^*\psi_\nu\right]M\psi_i d\tau \text{....................(3)}$$

M is dimension less matrix element, which is an operator. Neutrino interact weakly with nucleons, so we use time independent wave function. The free particle propagation constant $K = \frac{P_\nu}{\hbar}$ as

$$\psi_\nu = V^{-1/2}\exp\left[-\left(\frac{i}{\hbar}\right)p_\nu r\right] \text{...........................(4)}$$

For ejected electron

$$\psi_e^* = V^{-1/2}\ \exp\left[-\left(\frac{i}{\hbar}\right)p_e r\right] \text{..........................(5)}$$

here V is the volume in which the system for normalization purposes.

P_ν and P_e are the momenta of the neutrino and electron respectively, r is the position coordinate.

By assuming the plane waveform for the wave function of the electron and neutrino, we have neglected their possible interactions with the nucleus.

The matrix element becomes

$$H_{if} = g\int \psi_f^* \left\{ \frac{1}{v} \exp\left[-\frac{i}{\hbar}(p_e + p_v)r \right] \right\} M\psi_i d\tau \ldots\ldots\ldots\ldots(6)$$

The exponential factor can be written as

$$\exp\left[-\frac{i}{\hbar}(p_e + p_v)r \right] = 1 - \frac{i}{\hbar}(p_e + p_v)r - \frac{1}{2\hbar^2}[(p_e + p_v)r]^2 + \ldots\ldots\ldots\ldots(7)$$

Be neglecting higher order terms

$$H_{if} = \frac{g}{v}\int \psi_f^* M\psi_i d\tau = \frac{g}{v}\left| M_{if} \right| \ldots\ldots\ldots\ldots(8)$$

where $\left| M_{if} \right|$ is the overlap integral or the nuclear matrix element of the final and initial wave functions of the nucleus.

STATISTICAL FACTOR; The position and momentum of electron or neutrino can be represented by a point in phase space, the space containing three spatial and three momentum dimensions. The uncertainity principle prevents us from representing a moving particle by a single vector. This is because such a representation would amount to specifying both the position and momentum exactly . Thus phase space must be divided into cells of volume

$$\Delta x \, \Delta y \, \Delta z \, \Delta P_x \, \Delta P_y \, \Delta P_z$$

The number of states of a particle restricted to a volume V in actual space and whose momentum lies between the limits P and P+dP is given by

$$dN = V \times 4\pi P^2 \frac{dP}{h^3} \ldots\ldots\ldots\ldots(9)$$

The number of states corresponding to the appearance in volume V of the electron with the momentum in the range P_e to $P_e + dP_e$ is

$$dN_e = 4\pi V P_e^2 \frac{dP_e}{h^3} \qquad \text{.......................(10)}$$

$$\textit{Similarly} \quad dN_v = 4\pi P_v^2 \frac{dP_v}{h^3} \qquad \text{..............................(11)}$$

An electron and neutrino are independent of one another and Hence the number of states available to them jointly given by

$$dN = \left(4\pi V P_e^2 \frac{dP_e}{h^3}\right)\left(4\pi P_v^2 \frac{dP_v}{h^3}\right) \qquad \text{......................(12)}$$

The number of states per unit energy of the electron is

$$\frac{dN}{dE_0} = \frac{\frac{16\pi^2 V^2}{h^6} P_e^2 P_v^2 dP_e dP_v}{dE_0} \qquad \text{..........................(13)}$$

Total available energy

$$E_0 = E_v + E_e \qquad \text{...................(14)}$$

For fixed electron energy E_e, we have

$$d\,E_0 = dE_v \qquad \text{.......................(15)}$$

The momenta P_e and P_v are related to the electron and neutrino energy respectively by the eqn

$$E_0^2 = P_e^2 C^2 + m^2 C^4 \qquad \text{....................(16)}$$

$$E_v = CP_v \qquad \text{....................(17)} \qquad (\textit{assuming zero rest mass})$$

$$dE_v = CdP_v \qquad \text{....................(18)}$$

Using (15) ,(16), (17), (18) and (13) we have,

$$\frac{dN}{dE_0} = \frac{\frac{16\pi^2 V^2}{h^6} P_e^2 P_v^2 dP_e dP_v}{dE_0}$$

$$P_v = \frac{E_v}{c}$$

$$P_v = \frac{E_0 - E_e}{c}$$

$$P_v^2 = \frac{(E_0 - E_e)^2}{c^2}$$

Assuming zero rest mass , for fixed electron

Energy

$$dE_v = dE_0$$

$$dP_v = \frac{dE_v}{c} = \frac{dE_0}{c}$$

$$\frac{dP_v}{dE_0} = \frac{1}{c}$$

$$\therefore \frac{dN}{dE_0} = \frac{16\pi^2 V^2}{h^6} P_e^2 \left(\frac{E_0 - E_e}{c}\right)^2 dP_e \frac{1}{c} \quad \text{..........................(19)}$$

H_{if} *and* $\frac{dN}{dE_0}$ values are substitute in (1)

$$P(P_e)dP_e = \frac{2\pi}{\hbar} \left|H_{if}\right|^2 \frac{dN}{dE_0}$$

$$= \frac{2\pi}{\hbar} \frac{g^2}{V^2} \left|M_{if}\right|^2 \frac{16\pi^2 V^2}{h^6} P_e^2 \left(\frac{E_0 - E_e}{c}\right)^2 dP_e \frac{1}{c}$$

$$P(P_e)dP_e = \frac{g^2 \left|M_{if}\right|^2}{2\pi^3 c^3 \hbar^7} (E_0 - E_e)^2 P_e^2 dP_e \quad \text{..........................(20)}$$

Coulomb Correction: In the derivation of above relation no account has been taken of the coulomb interaction which can be neglected only for the lightest nuclei(Z<10) and sufficiently high electron energies. The plane wave for the emitted electron must be replaced by distorted coulomb wave function. This can be taken in to account by multiplying $|\psi_e|^2$ with a factor some time called coulomb factor $F(Z, E_0)$ also called Fermi function. It is the ratio of electron density at the daughter nucleus to the density at infinity i.e.

$$F(Z, E_e) = \frac{|\psi_e(0)|^2_{Coulomb}}{|\psi_e(0)|^2_{Free}} \text{..........................(21)}$$

In non-relativistic approximation, it has the value

$$F(Z, E_e) = \frac{2\pi\eta}{(1 - e^{-2\pi\eta})} \text{....................(22)}$$

where $\eta = \frac{Ze^2}{4\pi\varepsilon_0} \mathrm{h}\upsilon$ for electrons, $\eta = -\frac{Ze^2}{4\pi\varepsilon_0} \mathrm{h}\upsilon$ for positrons

'Z' the atomic number of the product nucleus and V being velocity of electron at a great distance from the nucleus. When consideration is given to this affect equation 20 becomes

$$\begin{aligned} P(P_e)dP_e &= \frac{g^2 |M_{if}|^2}{2\pi^3 c^3 \hbar^7} F(Z, E_0)(E_0 - E_e)^2 P_e^2 dP_e \\ &= c^2 F(Z, E_0)(E_0 - E_e)^2 P_e^2 dP_e \text{.........................(23)} \end{aligned}$$

$$c = g |M_{if}| \left[2\pi^3 c^3 \hbar^7\right]^{-1/2} \text{...........................(24)}$$

The coulomb correction enhances the probability of electron emission and decreases the probability of positron emission . At low energies, the coulomb force loses its effect at high energies.

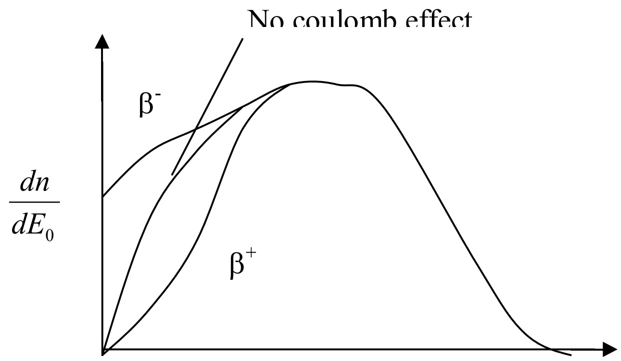


Fig. 7.1. Theoretical β - energy spectrum (Fermi theory)

7.5.SELECTION RULES: If electron and neutrino are emitted with their intrinsic spins antiparallel (singlet state), the change in nuclear spin must be strictly zero, if these are emitted with their spins parallel (triplet state) ΔI may be $+1, 0 -1$. The former selection rule was proposed by Fermi and the latter was suggested by Gamow and Teller. Both types of allowed transitions orbital angular momentum and parity unchanged electron and neutrino carry away no orbital angular momentum.

The allowed transitions of the type $\Delta I = 1$ obeying G – T selection rule and forbidden for F-selection rule.

The allowed transitions of the $0 \rightarrow 0$ type that are allowed by F- selection rule. But forbidden by G-T selection rule.

Thee allowed transitions are further classified as favoured (super allowed) unfavoured transitions . The allowed transitions is favoured if the nucleon which changes its chare remains in the same level, it is unfavoured if the nucleons changes its level, most of the allowed β - transitions are unfavoured.

FIRST FORBIDDEN; For these transitions $I_\beta = 1$ and parity changes

Fermi selection rule: $\Delta I = \pm 1, 0$ (except $0 \rightarrow 0$)

Gamow Teller rules: $\Delta I = \pm 2, \pm 1, 0$ (except $0 \rightarrow 0, 1/2 \rightarrow 1/2, 0 \leftrightarrow 1$)

Example: $Kr^{87} \longrightarrow Rb^{87} + \beta^- \ (5/2 \rightarrow 3/2)$

$Ag^{111} \longrightarrow Cd^{111} + \beta^- \ (1/2 \rightarrow 1/2)$

$Ce^{141} \longrightarrow Pr^{141} + \beta^- \ (7/2 \rightarrow 3/2)$

SECOND FORBIDDEN; For these transitions $I_\beta = 2$ and no change in parity.

Example:

$Cs^{135} \longrightarrow Ba^{135} + \beta^- \ (7/2 \rightarrow 3/2)$

$Be^{10} \longrightarrow B^{10} + \beta^- \ (0 \rightarrow 3)$

$Na^{22} \longrightarrow Ne^{22} + \beta^+ (3 \rightarrow 0)$

Fermi selection rules: $\Delta I = \pm 2, \pm 1$, (except $0 \leftrightarrow 1$)

Gamow Teller rules: $\Delta I = \pm 3, \pm 2, 0 \rightarrow 0$ (except $0 \leftrightarrow 2$)

n^{th} forbidden:

Fermi selection rule $\Delta I = \pm n, \pm (n-1)$ parity changes for n odd

G-T selection rule $\Delta I = \pm n, \pm (n+1)$ parity does not change for n even.

Keywords:
Neutrino hypothesis, electron capture, over lap integral

Self assessment questions:

1. What are the three forms of beta decay.
2. How does the neutrino hypothesis over come the difficulties that are experienced with beta ray spectrum.
3. Explain in detail about the Fermi theory of beta decay.
4. Explain the selection rules of beta decay.

Text books

1. Nuclear physics by D.C.Tayal , Himalaya publishing company,Bombay.
2. Nuclear physics by R.C.Sharma, K.Nath&co, Merut
3. Nuclear physics by S.B.Patel

Unit 2

Lesson 8

CLASSIFICATION OF ELEMENTARY PARTICLES

The objectives of the lesson are to explain the following:

8.1 Introduction

8.2. Classification of Elementary Particles

8.3 Interaction between the elementary Particles

8.4 Conservation Laws

8.5.Charge Conjugation

8.6.Space-Inversion Invariance (Parity)

8.7.Combined Inversion (CP)

8.8.Time Reversal Symmetry and CPT Invariance

8.9.Summary

8.1. Introduction: Extensively researches, have been carried out by the scientists to conclude about the ultimate representatives of the matter that may be the basic building blocks-nowadays called as elementary particles. A summary of such works is given below:

a) In the beginning of nineteenth century, it was established that matter is composed of atoms and molecules. But soon it was found that atom has also a rich structures and in 1897, J.J. Thomson established the existence of a particle-the electron that still is classified as an elementary particle.

b) At the start of twentieth century experiment and ideas of Rutherford and Bohr established that atom consisted of a positively charged nucleus with electrons revolving around it.

c) In 1932, J.Chadwick identified neutron and W. Heisenberg suggested that atomic nuclei consist of neutrons and protons. Thus atomic picture becomes somewhat clear with electron, neutron, proton and photon as the basic building blocks, photon has been added as a field particle for electromagnetic forces such as exist between the nucleus and electrons in the atom, i.e., it is a quantum unit of radiation. It has zero rest mass and is uncharged.

In the some year, C.D. Anderson found the positive electron or the positron while studying cosmic-ray showers. The discovery of this particle, being the antiparticle of electron, predicted the existence of antimatter. With this discovery it was thought that the atomic picture could be completed, apart from four afore-said particles with three possible antiparticles—antielectrons, antiprotons, and antineutrons;

8.2. Classification of Elementary Particles:

The elementary particles are separated in to two general groups called bosons and Fermions . These two groups have different types of spin and their behaviour is controlled respectively by a different kind of statistics

a) Bose Statistics

b) Fermi Statistics

Bosons are particles with intrinsic angular momentum equal to an integral multiple of $\hbar$. Fermions are all those particles in which the spin is half integral. The most important difference between the two classes of particles is that there is no conservation law controlling the total number of bosons in the Universe, where as the total number of fermions is strictly conserved.

Bosons is a term, which not only includes material particles but also includes those quanta and Photons which arise from interactions. Thus in the case of the simple electromagnetic field the bosons are merely the light photons or the X-ray photons. The photon has a mass of zero and a spin of unity and consequently described as a massless boson. A massless boson, called a graviton with a probable spin of two units has been postulated as a field particle for gravity. These bosons, created by the electromagnetic field, are essentially of one kind, while the bosons formed in the strong interaction are of two distinct kinds. First there are those which are known as pions or π-mesons (π^+, π^-, π^0). The second group of bosons are much heavier than that of pions, and are known as kaons or K-mesons (K^+,K^-,K^0).

The fermions fall in two main classes, according to whether they are lighter than mesons, or heavier. Those in the heavier group are called baryons. The leptons are the electrons, muons and neutrinos and their anti-particles. There are all with mass less than the pions and with spin half. Leptons interact weakly with other particles. The total number of leptons minus the total number of anti-leptons remains unchanged in all reactions and decay processes involving leptons and anti-leptons and decay process involving leptons and anti-leptons. The baryons consist of the two nucleons with their anti-particles ($n^0, \overline{n}^0 : p^+, p^-$) and the hyperons. Hyperons are the extremely unstable some what heavier particles and can be divided in to four sub-groups - particles (a neutral particle of mass about 180m_e) The Σ - particles (Σ^- , Σ^0 & Σ^+ with masses in the range 2320 to 2340m_e) the Ξ – particles (Ξ^-, and Ξ^0 with masses near 2580 m_e) and the $\overline{\Omega}$ particle (of mass about 3284 m_e). There is no reason to doubt the existence of the anti-particles of these fermions. The total number of baryons minus the total number of anti-baryons is absolutely conserved in all interactions. The kaons and pions together with the barons are placed in to group of strongly interacting particles, called hardons.

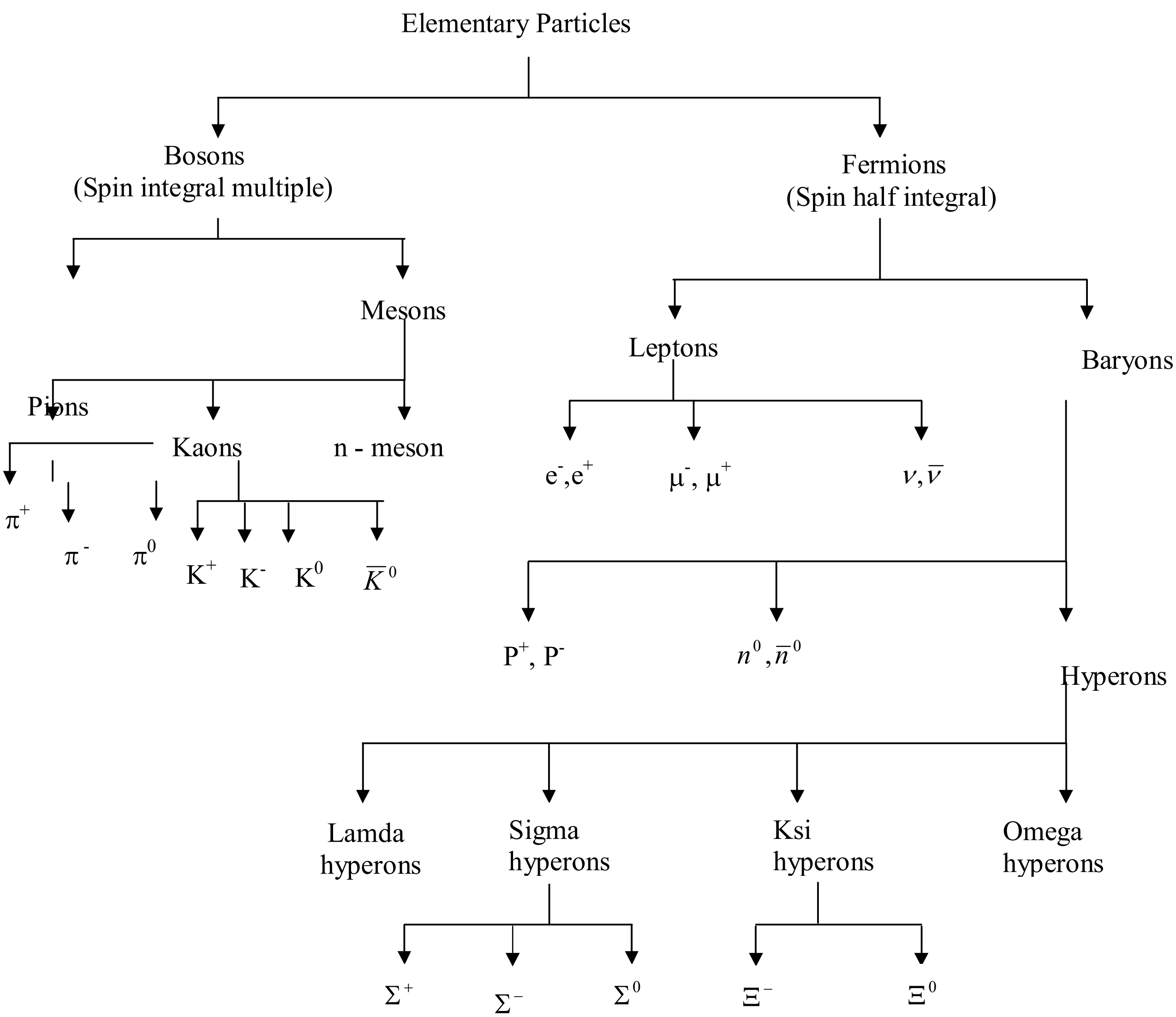


Fig. 8.1.Classification of elementary particles

8.3. INTERACTION BETWEEN THE ELEMENTARY PARTICLES:

The interactions among elementary particles can be classified in to following four types.

1.The gravitational Interaction

2.Electromagnetic Interaction

3.Strong Interaction

4.Weak Interaction

1.The gravitational Interaction: The first force that any of us discover is gravity. It holds the moon and earth together, keeping the planets in their solar orbits and binds stars to form our galaxy. Newton gave a formula $F = G\frac{m_1 m_2}{r^2}$ for the interaction between two masses. The gravitational effect does not depend on the colour, size , charge, velocities, spin and angular orientation but depends on the magnitude of the inertia. The gravitational force between two nucleons separated by a nucleon diameter is $F = G\frac{m_1 m_2}{r^2}$ $= 6.7 \times 10^{-11}\frac{(1.7\times 10^{-27})^2}{(10^{-15})^2} \approx 2 \times 10^{-34}$ newton. And the gravitational attraction is only about 2×10^{-49} joule. Hence we see that it plays no role in particle reactions. Gravitation can thus be explained in terms of the interactions of gravitons. Their mass must be zero and therefore . Their velocity must be that of light. As the gravitational field is extremely weak. The gravitons can not be detected in laboratory.

2.Electromagnetic Interaction: The term electromagnetism is because the electricity and magnetism are both part of the same phenomenon. According to coulomb's law $F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$ For two protons 10^{-15} meters apart.The repulsion force will be $9\times 10^9 \times \frac{(1.6\times 10^{-19})^2}{(10^{-15})^2} \approx$ 30 newtons. It is about 10^{-35} times greater than the gravitational attraction caused by the mass. The energy released by the complete seperation of these protons would be 3×10^{-14}Joules.

If the particles are not at rest but are moving, the field will not only be an electric field but would be new one depending on the velocity and magnitude of the charge. When the charge is accelerated, the energy is radiated out in the form of an electric and magnetic pulses. The pulse is called a photon and travels with the velocity of light. The interaction between two charged particles consists of an exchange of these photons.

The electromagnetic interaction is charge. Dependent. The electromagnetic interaction violate the law of isospin conservation. All other quantities such as charge parity, baryon number, lepton number, hypercharge, strangeness number are conserved.

Other examples of such type of interaction are the pair formation from photon and vice versa and the decay of neutral pion in to two gamma ray photons.

$$\pi^0 \longrightarrow r + r$$
$$\eta^0 \longrightarrow r + r$$

3.STRONG INTERACTION: The strong nuclear interaction is independent of the electric charge. The force is same between p- p & n-n . For this purpose the proton and neutron are one but in different electric charge states. Strong interactions involves mesons and baryons, The range is very much shorter than that of gravitational or electromagnetic interaction. Strong interaction falls off rapidly when the distance between two particles increases. Yukawa predicted that the new particles called mesons, should have a mass of the order of 200 to 300 electron masses.

The strong interaction between elementary particles are responsible for the total cross section as a function of energy. The strong interaction is a short range force ($\approx 10^{-15}$ m) and it conserves baryon number b, charge Q, hypercharge Y, parity Π, isospin T and its component T_z. It is responsible for kaon production, however the decay of mesons, nucleons and hyperons proceeds by an electromagnetic or weak interaction.

4.WEAK INTERACTION: The weak interaction is responsible for the decay of strange and non-strange particles and for non-leptonic decays of strange particles. The numerical constant which is characterstic of the weak interaction is obtained from Fermi's theory of β- decay. Its value is $g_F = 1.41 \times 10^{-62}$ Jm^2. In analogy with the expression for the other interactions the dimensionless weak interaction coupling constant is of magnitude $\frac{g_F^2}{(\hbar c)^2} \left(\frac{m_\Pi c}{\hbar}\right)^4 = 5 \times 10^{-14}$

Consider the reaction which do not involve a change of strangeness and yet which must be due to weak interaction. The neutron decay is the proto-type of all the β- decay.

$$n \longrightarrow P + e^- + \bar{\upsilon}_e$$

comparison of the four basic interactions

Field	Relative magnitude	Associated particles	Characteristic time
Strong interaction	1	Pion, kaon	10^{-23} sec
Electromagnetic Interaction	10^{-3}	Photon	10^{-20}sec
Weak interaction	10^{-13}	Intermediate boson	10^{-10}sec
Gravitational	10^{-39}	Graviton	10^{6}sec

8.4. CONSERVATION LAWS: The behaviour of the elementary particles is restricted by a number of conservation laws or invariance principles. That is to say, certain properties of representative physical quantities must remain unchanged in any process. The most familiar quantities in large scale experiments that are conserved in all interactions.

1.Conservation of Linear momentum: The linear momentum of a body is defined as product of its mass and velocity P = mv. When the net external force acting on the system is zero. The total Linear momentum of a system remains constant. i.e. $\frac{dP}{dt} = 0$, P = Constant

2.Conservation of Angular momentum: The conservation of angular momentum includes both orbital and spin angular momentum together. The first is given by the motion of the object as a whole about any chosen external axis of rotation. The second is the intrinsic angular momentum of each about an axis through its own centre of mass.

3.Conservation of Energy: Conservation of energy on other hand seems more complicated with elementary particles because a large fraction of the total energy is oftenly interchanged between rest energy associated with mass and kinetic or potential energy. The sum of these three, the total energy is always conserved in any reaction.

$$\text{Ex:-}\quad K^0 \longrightarrow \pi^+ + \pi^- + \pi^0$$

The rest energy K^0 particles enough to produce π^+, π^-, π^0 particles.

4. Conservation of Charge: The most familiar of the conservation laws is the conservation of electric charge. The Charge is conserved in all process i.e. the total charge remains unchanged.

5. Conservation of Lepton number: The leptons are the fundamental particles that interact only through weak and electromagnetic interaction. The number of Leptons minus the number of anti-Leptons is conserved. The ordinary electron. Negative muon and neutrinos all have a lepton number +1. the corresponding anti leptons have a lepton number is –1.

$$\text{Ex:-}\quad \mu^+ \longrightarrow e^+ + \nu + \bar{\upsilon}$$

$$-1 = -1 + (-1) + 1$$

$$-1 = -1$$

This reaction is allowed

6.Conservation of Baryon number: The number of baryons minus the number of anti-baryons is conserved. In other words the net baryon number in any process always remain unchanged

In other word the net baryon number in any process always remains unchanged. All normal baryons such as P^+, n^0,π^0 Σ^+, Σ^0, Ξ^-, Ξ^0 and Ω have baryon number is +1. Anti barons have –1.

Ex:- $\pi^0 \longrightarrow \pi^+ + P^-$

$1 = 0 + 1$

$\Delta B = 0$ so this process is allowed.

7. CONSERVATION OF ISOSPIN: According to the ordinary idea of isotopic spin, each nuclear particle possess a certain total isotopic spin T and each possible projection of this isotopic spin along a certain axis T_3 appears to us as a different charge state of the corresponding particle.

In the case of nucleons, T = ½ and the 2T +1 i.e. $2\frac{1}{2}+1=2$, the possible values of T_3 are +1/2 (for proton state), -1/2 (for neutron state)

The Isospin of nucleons is ½.

For pions T = 1 and 2T + 1 = 2(1) + ! = 3

The possible values of T_3 are +1 (for π^+)

0 (for π^0)

-1 (for π^-)

The Isospin of pions is 1.

Relation between the quantities T_z, B and Q are

$$Q = T_z + B/2$$

Here B is the baryon number. T is the isotopic spin quantum number, T_z the component of T and Q is charge. B is+1 for proton, neutron & hyperon

B is zero for pion.

$Q_p = +1/2 + ½ = 1$

$Q_n = -1/2 +1/2 = 0$

$Q_{\pi^+} = +1 +0 = 1$

$Q\ \pi^0 = 0 + 0 = 0$

$Q_{\pi^-} = -1 + 0 = -1$

Isospin numbers are associated with hardrons (particles that exhibit strong interactions) but not with leptons. The isospin component T_z is conserved in both strong and electromagnetic interaction but not in weak interaction.

8.CONSERVATION OF HYPERCHARGE: A quantity called hypercharge is also conserved in strong and electromagnetic interaction. For example, for the triplet π^+, π^-, π^0. average charge is zero and hence all these three mesons have a hypercharge of zero.The hyper charge of the pair particles K^+ and K^0 is +1 and that of the pair of antiparticles K^- and K^0 is -1. Thus the alternative definition is that it is twice the difference between the actual charge Q and the isospin component T_z of a particle. Thus hyper charge

$$Y = 2(Q - T_z)$$

9.CONSERVATION OF STRANGENESS: The concept of strangeness has found wide application in particle physics. It is an additional quantum number which describes the interactions of elementary particles. It has been chosen in such a manner that it becomes zero for all the well know particles(non-strange particles)

One of the most common V-particles (π^0) was neutral and decayed ($\pi^0 \longrightarrow P + \pi^-$) in times 2.5×10^{-10} seconds. The V-particles can interact strongly and therefore, are produced only in pairs, once separated each number can decay in to ordinary particles only through the weak interaction. A typical example is

$$\pi^- + p^+ \longrightarrow \Lambda^0 + k^0 \longrightarrow \pi^+ + \pi^-$$
$$\downarrow \longrightarrow p^+ + \pi^-$$

Both the associated creation of the strange particles and their individual stability against immediately decay were the features that earned than the title strange. Both features can be explained by insisting that the total strangeness must remain constant in fast particle reaction.

If the baryons are arranged in columns according to their electric charge. The electric charge centre of the nucleon is at +1/2 half way between P and n. The charge centre of π^0 is at 0. The triplet sigma is centred at 0 , but the doublet X_i is centred at -1/2. The -- singlet is at charge -1. If we take charge centre of the nucleon doublet arbitrarily too be reference origin. Then we have

$$\text{For } \Lambda^0 ....\Delta Q = Q_\Lambda - Q_N = -\frac{1}{2};\ \ for\ \ \Sigma - hyperon...\Delta Q = Q_\Sigma - Q_N = -\frac{1}{2}$$

$$\text{For } \Xi \text{ doublet}\ldots \Delta Q = -1\ \ for\ \ \overline{\Omega}....\Delta Q = -\frac{3}{2}$$

By defining strangeness Quantum number as

$$S = 2\Delta Q$$

we obtain S = 0 for the nucleons and non-zero for hyperons.

Ex:-

$$\Sigma^+ \longrightarrow \Lambda^0 + e^+ + v_e$$
$$\Delta S = 0$$
$$\Sigma^- \longrightarrow n + e^- + \overline{v}_e \Rightarrow \Delta S = 1$$
$$\Lambda^0 \longrightarrow P + e^- + \overline{v}_e \Rightarrow \Delta S = 1$$

Baryons:

Table

Particle	Symbol	Anti-Particle	Strangeness
Proton	P	$\bar{p}$	0
Neutron	N	$\bar{n}$	0
Lambda	Λ^0	$\bar{\Lambda}^0$	-1
Sigma	Σ^+	$\bar{\Sigma}^-$	-1
	Σ^0	$\bar{\Sigma}^0$	-1
	Σ^-	$\bar{\Sigma}^+$	-1

8.5. CHARGE CONJUGATION: Charge conjugation is defined as the interchange of particles and anti particles. It does not simply mean a change over the opposite electric charge or magnetic moment, the sign of other charge quantum numbers (hypercharge Y, baryon number B, lepton number (l_e, l_μ) is also reversed with out charging mass m and spin s Thus a unitary operator, also known as charge conjugation operator C satisfies the following relation.

$$CQC^{-1} = -Q, \quad CYC^{-1} = Y, \quad CBC^{-1} = -B$$
$$Cl_eC^{-1} = -l_e, \quad Cl_\mu C^{-1} = l_\mu$$

Some elementary particles e.g., r, π^0 - mesons and the positronium atom $(e^+ + e^-)$ are transformed in to themselves by charge conjugation. They are their own anti-particles. These are known as self conjugate or true neutral particles. The neutron (B= 1, Y = 1) and K^0 – mesons (Y =1 , B = 0) are not invariant under C.

8.6.SPACE – INVERSION INVARIANCE (PARITY):- The parity principle says that there is a symmetry between the world and its mirror image. This may be defined as reflection of every point in space through the origin of a co-ordinate systems $x \rightarrow -x$, $y \rightarrow -y$, $z \rightarrow -z$. If a system or process is such that its mirror image is impossible to obtain in nature. The system of process is said to violate the law of parity conservation.

All phenomena involving strong and electromagnetic interactions alone do conserve parity. In these cases the systems can be classified by the eigen values of the parity operator P. For a single particle schrodinger wave function Ψ, the result of the parity operator is

$$P|\psi(x)\rangle = e^{i\alpha}|\psi(+x)\rangle$$

As α is an arbitrary real phase, hence can be set equal to zero.

$$P|\psi(x)\rangle = |\psi(-x)\rangle$$
$$P^2|\psi(x)\rangle = |\psi(x)\rangle$$

It shows eigen values of P as +1 or -1

The parity of the photon depends upon the mode of transition, it is due to the charge of the sign of electromagnetic current under the parity operation. The nucleons and electrons are assigned positive or even intrinsic parity. The pions have negative or odd parity as they involve in strong interactions with nucleons. K – mesons and n^0- mesons have negative parity. $\Lambda^0, \Xi^-, \Sigma, \Omega$ hyperon have positive intrinsic spin.

The consnervation of parity is applicable only to strong and electromagnetic interaction but not weak interaction.

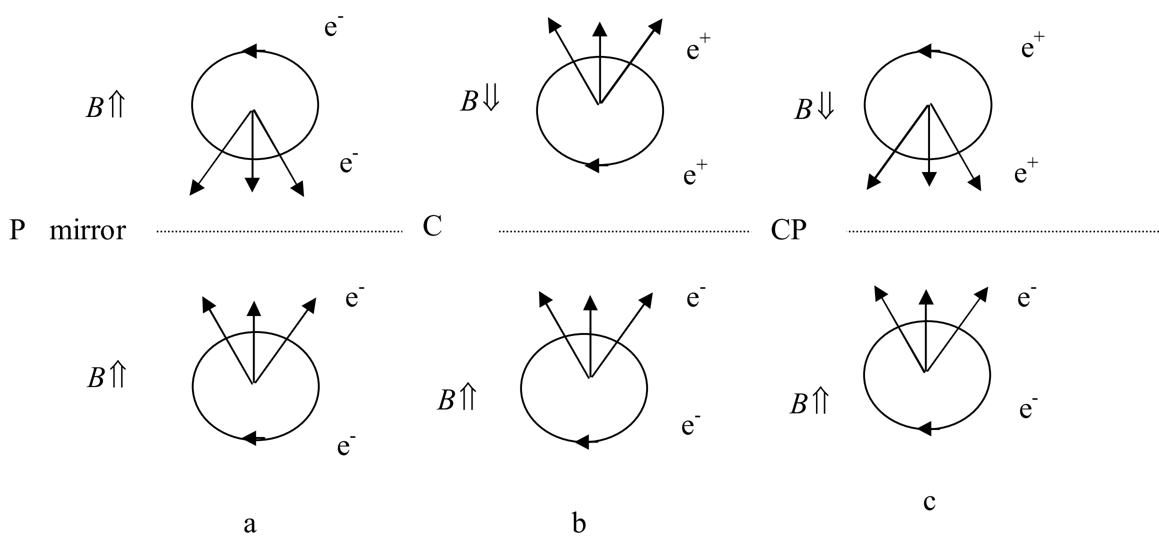
8.7.COMBINED INVERSION(CP): Landu (1956) advanced a hypothesis to the effect that any physical interaction must be invariant under simultaneous reversal of position coordinates and change over from particle to anti-particle. For example a neutrino has a definite helicity and its parity conjugate has opposite helicity. The charge conjugate of the neutrino also has opposite helicity. Thus under the combined operation CP (or pc) the neutrino changes to anti-neutrino. The combined operation also known as combined parity (charge and space) is conserved in most of physical processes. Let us consider the decay of the positive pion

$$\pi^{+} \longrightarrow \mu_{L}^{+} + \nu_{\mu} L$$

Here subscript L indicates that neutrino and +ve meon fly apartr with left handed spin. S the C-inversion changes particles in to anti-particles and vice-versa, where as P-inversion converts left handed motion to right handed motion. Hence

$$C-inversion: \quad \pi^{-} \longrightarrow \mu_{L}^{-} + \nu_{\mu} L \quad impossible \quad process$$
$$P-inversion: \quad \pi^{+} \longrightarrow \mu_{R}^{+} + \nu_{\mu} R \quad impossible \quad process$$
$$CP-inversion: \quad \pi^{-} \longrightarrow \mu_{R}^{-} + \nu_{\mu} R \quad possible \quad process$$

Let us consider the case of the Beta decay of polarized nuclei. The interpretation of the parity non-conversation, charge non-conversation and conservation under combined operation is shown in fig.8.2.In this figure B shows the direction of a magnetic field due to current loop, used for polarizing the nuclei. It represents the nuclear spin and thus known as polarization vector.



a) Parity mirror b) Charge Conjugation mirror c) CP mirror

Fig. 8.2

The upper diagrams represent the result of the reflection of process shown in the lower diagram of fig:

8.8.TIME REVERSAL SYMMETRY AND CPT INVARIANCE: Time reversal symmetry, denoted by T, means that events an atomic or subatomic, or in general, on a particle scale should be exactly reversible. Basic postulates of quantum field theory combined with relativity require that all interaction should be invariant under at the combination of three operation C,P and T i.e. the CPT operation. Lee and Yang realised that in weak interactions of the type of Beta decay, P and C invariance are violated but CPT invariance is valid.

8.9.Summary: The elementary particles are separated in to two general groups called bosons and Fermions . These two groups have different types of spin and their behaviour is controlled respectively by a different kind of statistics

b) Bose Statistics

b) Fermi Statistics

Bosons are particles with intrinsic angular momentum equal to an integral multiple of $\hbar$. Fermions are all those particles in which the spin is half integral. The most important difference between the two classes of particles is that there is no conservation law controlling the total number of bosons in the Universe, where as the total number of fermions is strictly conserved.

The interactions among elementary particles can be classified in to following four types.

1.The gravitational Interaction

2.Electromagnetic Interaction

3.Strong Interaction

4.Weak Interaction

The behaviour of the elementary particles is restricted by a number of conservation laws or invariance principles. That is to say, certain properties of representative physical quantities must remain unchanged in any process. The most familiar quantities in large scale experiments that are conserved in all interactions.

Keywords:
Elementary particles, boson, fermion, meson, graviton

Self assessment questions:

1. Explain in detail about the classification and properties of elementary particles
2. What are various interactions that exist among the elementary particles.
3. Explain various conservation laws that are obeyed by elementary particles.
4. Explain the terms namely charge conjugation, space inversion invariance and combined inversion.

Text books

1. Nuclear physics by D.C.Tayal , Himalaya publishing company,Bombay.
2. Nuclear physics by R.C.Sharma, K.Nath&co, Merut
3. Nuclear physics by S.B.Patel